

# Probabilistic Databases under Updates

Boolean Query Evaluation  
and Ranked Enumeration

Christoph Berkholz, Maximilian Merz

Talk at AIMoTh 2022, based on the PODS 2021 paper

# 1 Probabilistic Databases under Updates

Boolean Query Evaluation  
and Ranked Enumeration

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# 1 Probabilistic Databases 2 under Updates

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# 1 Probabilistic Databases

## 2 under Updates

### 3 Boolean Query Evaluation and Ranked Enumeration

Christoph Berkholz, Maximilian Merz

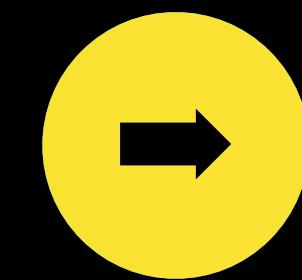
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# 1 Probabilistic Databases 2 under Updates

- 3 Boolean Query Evaluation
- 4 and Ranked Enumeration

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# Probabilistic Databases

②

## under Updates

③

Boolean Query Evaluation  
and Ranked Enumeration

④

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R	A
a	
b	

S	A	B
a	b	
a	d	

?

R	A	S	A	B
a		a	b	
b		a	d	

The diagram illustrates a data structure or mapping between two sets of data. It consists of two 2x2 tables, each with a black background and white borders, set against a black background.

**Left Table:**

R	A
a	b

**Right Table:**

S	A	B
a	b	
a	d	

A yellow arrow points from the question mark to the empty cell in the bottom-right corner of the right table.

Question:

?

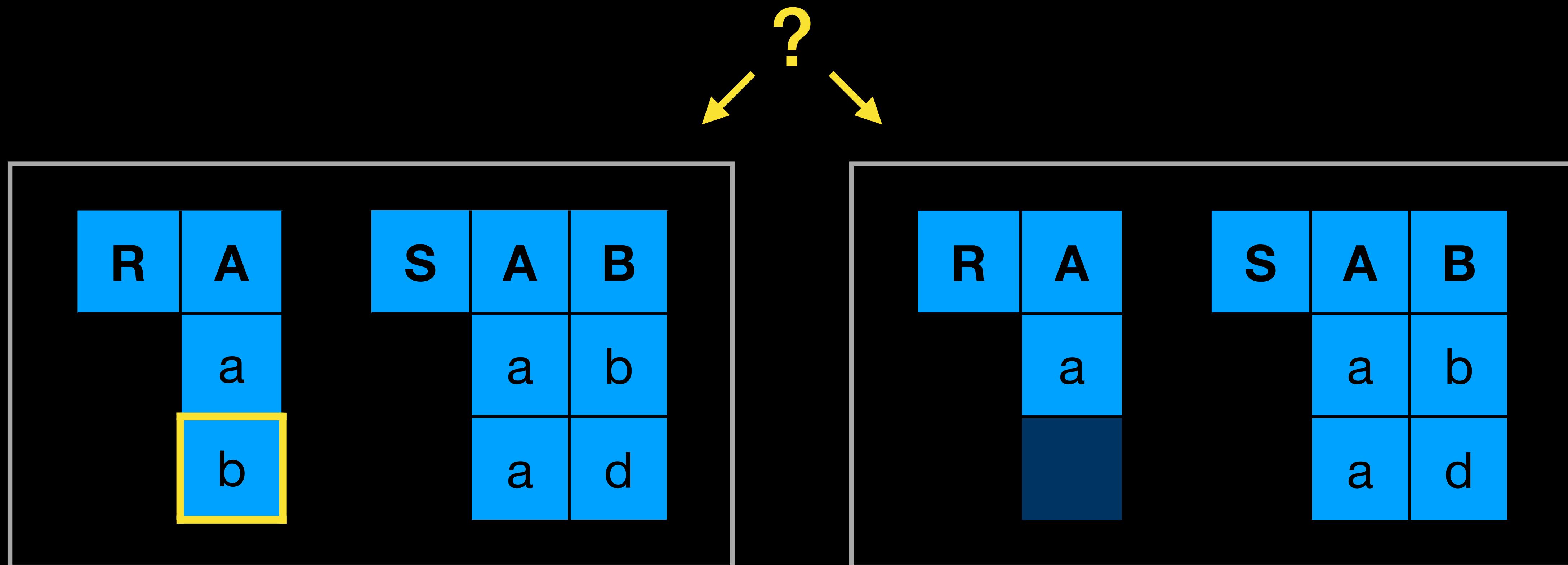
R	A
a	b
b	

?

↙ ↘

R	A
a	b
a	d

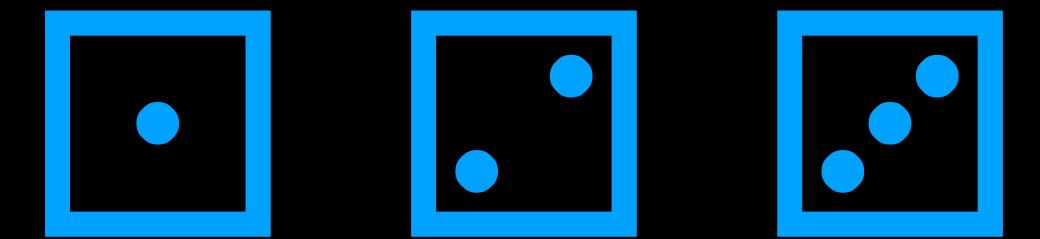
# Probabilistic Databases (PDBs)



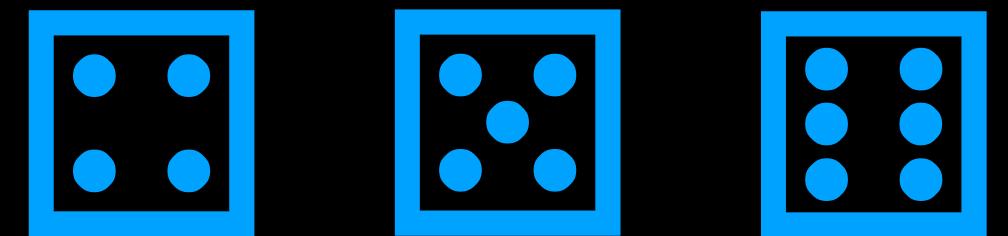
# Probabilistic Databases (PDBs)

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Dice roll 



Probability Space  
("possible worlds")

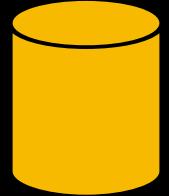


# Probabilistic Databases (PDBs)

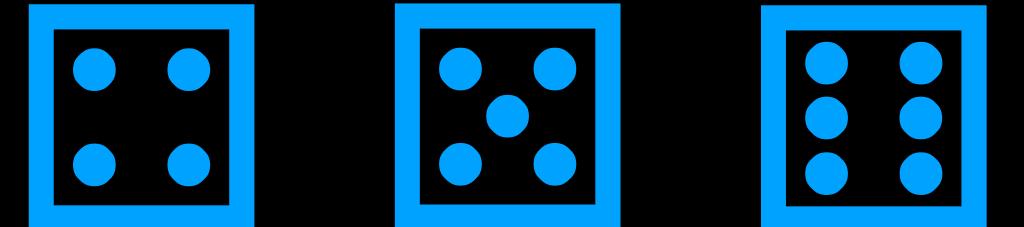
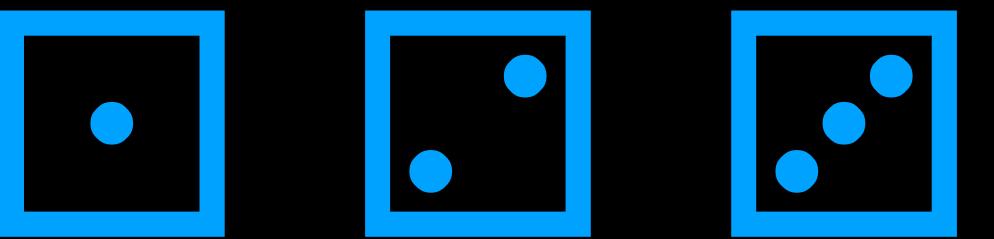
Dice roll



PDB



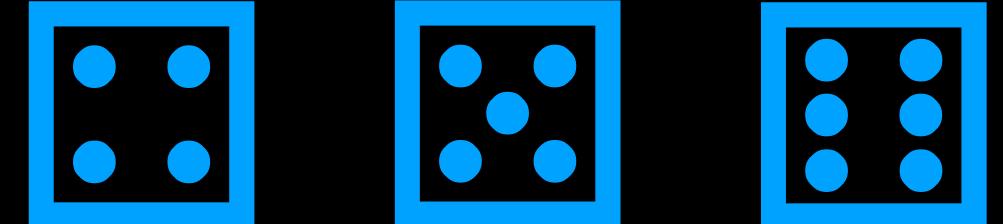
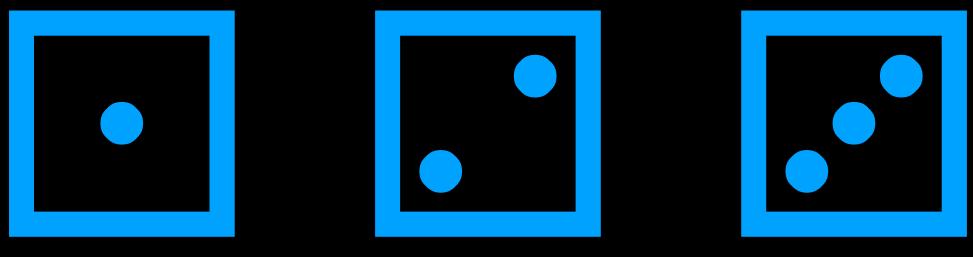
Probability Space  
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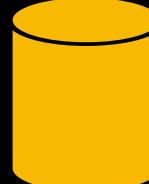


# Probabilistic Databases (PDBs)

Probability Space  
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Dice roll 



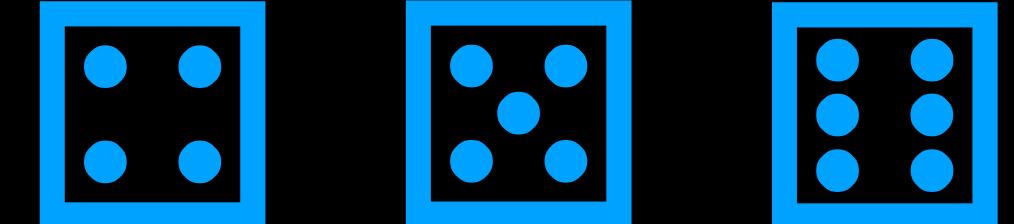
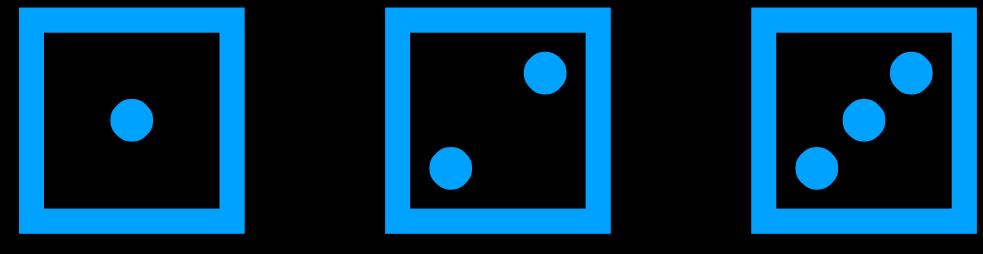
PDB 

R	A
a	
b	

# Probabilistic Databases (PDBs)

Probability Space  
("possible worlds")

Dice roll 



PDB 

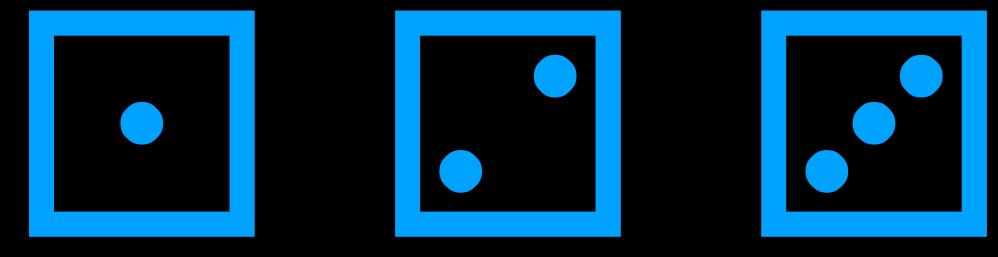
R	A
a	
b	

R	A
a	

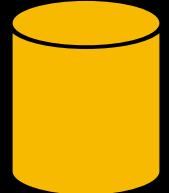
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Probability Space  
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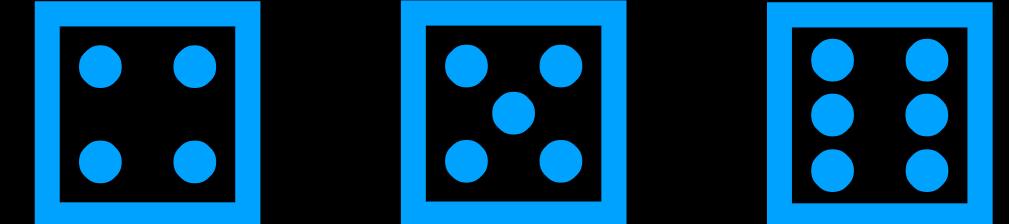


PDB



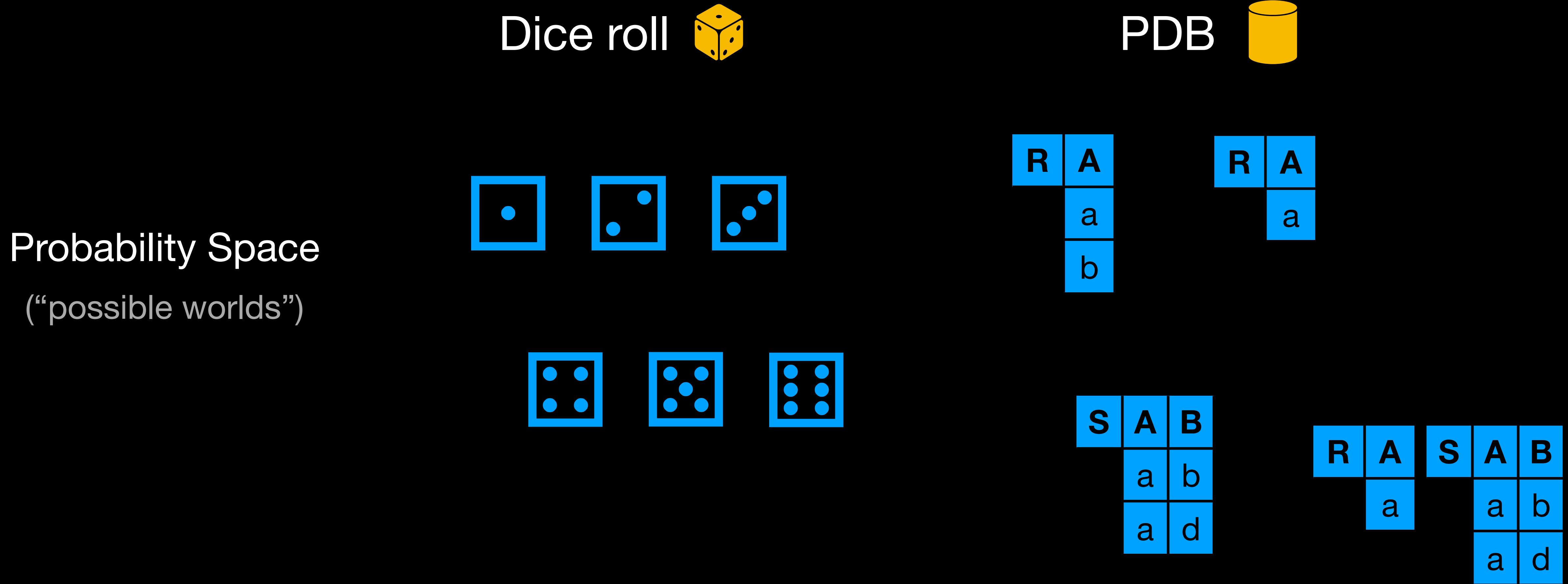
R	A
a	b

R	A
a	

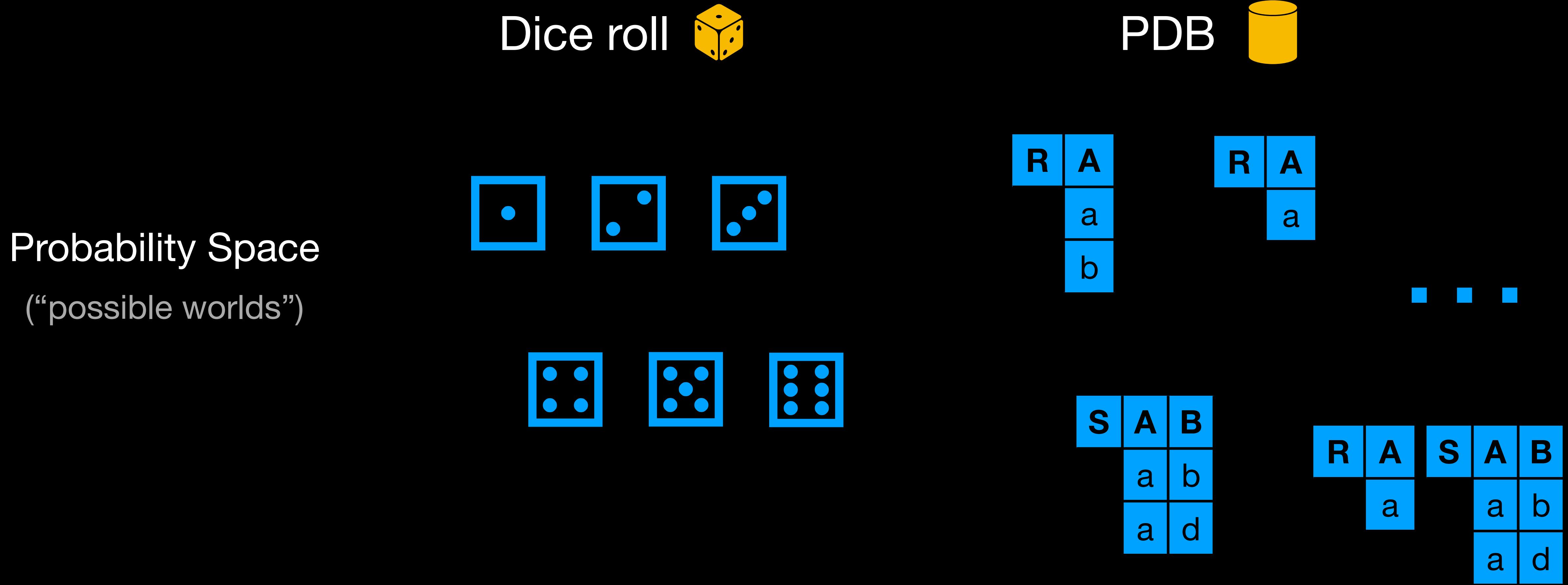


S	A	B
a	b	
a	d	

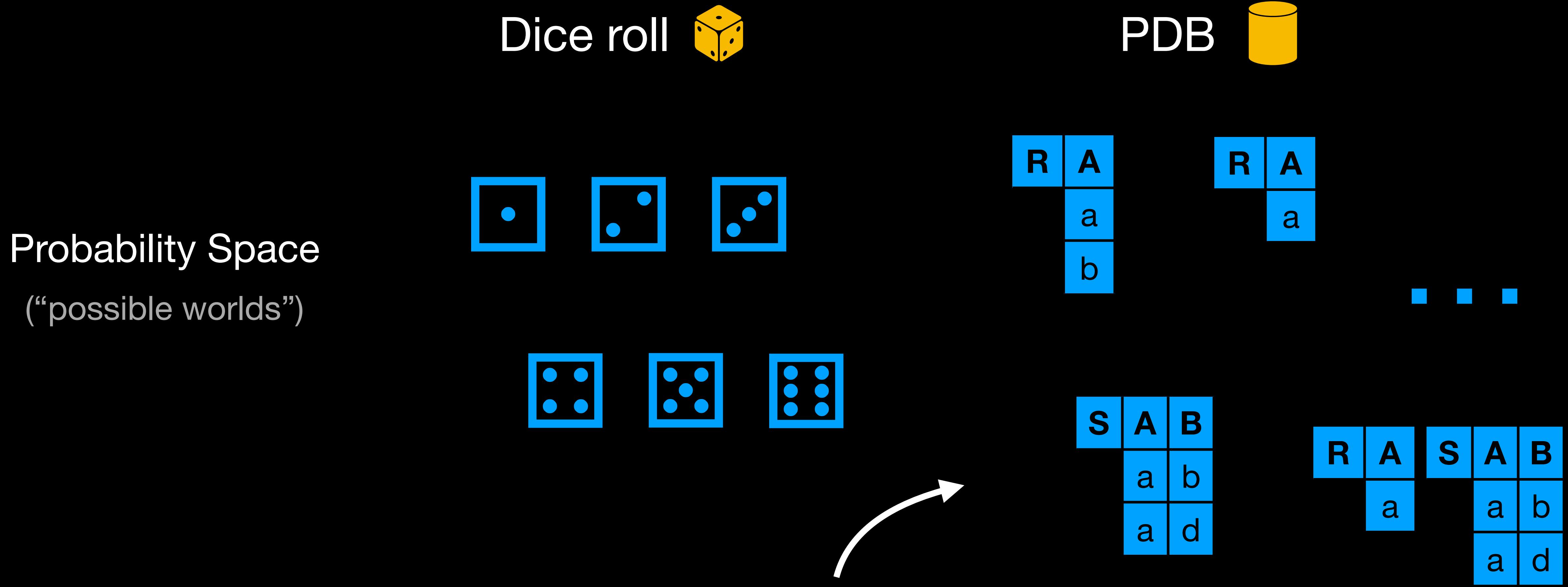
# Probabilistic Databases (PDBs)



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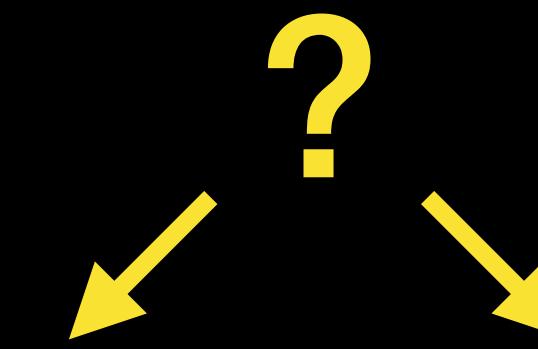


# Probabilistic Databases (PDBs)



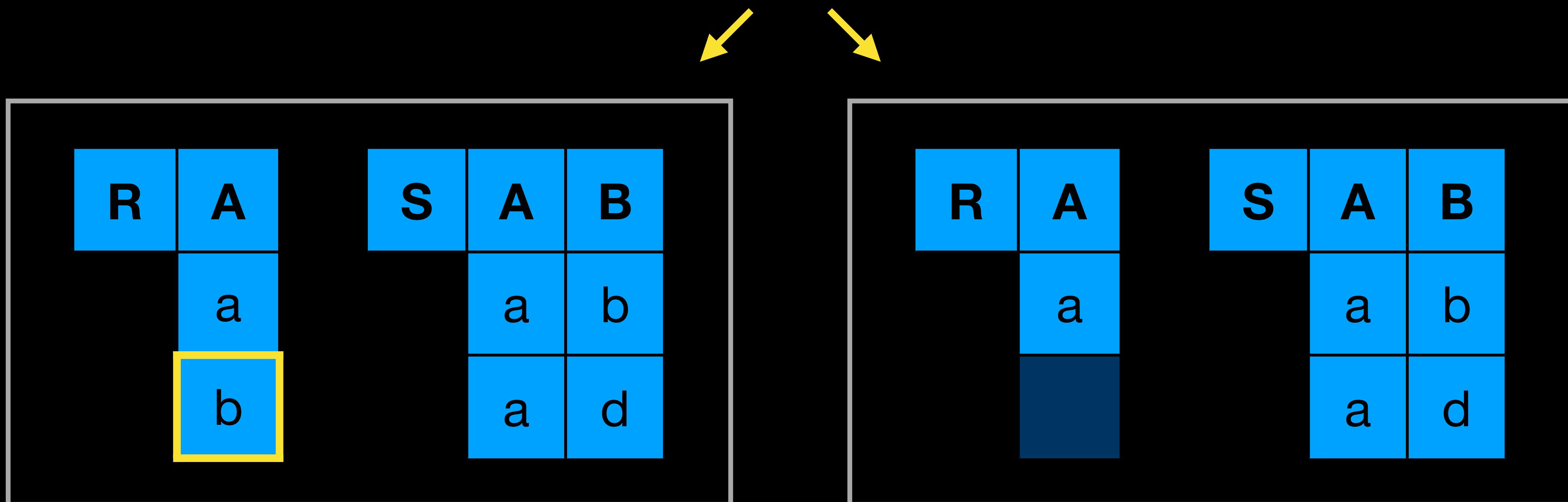
storing the set of possible worlds explicitly  
is not practical!

R	A
a	b
b	



R	A
a	b
a	d

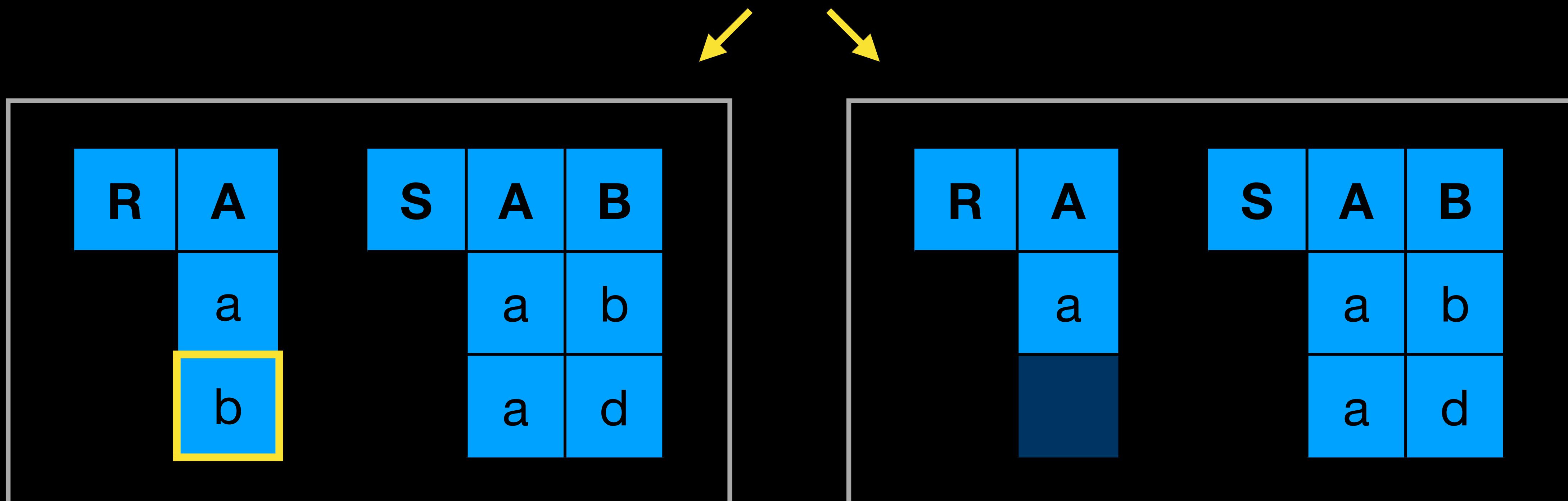
# Tuple-Independent PDB



# Tuple-Independent PDB

Assumption:

Every tuple is an independent probabilistic event



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Assumption:

Every tuple is an independent probabilistic event

R	A	p	
a	1		
b	$\frac{1}{3}$		
S	A	B	p
a	b	1	
a	d	1	



R	A	
a		
S	A	B
a	b	
a	d	

R	A	
a		
S	A	B
a	b	
a	d	

# Tuple-Independent PDB

Assumption:

Every tuple is an independent probabilistic event

R	A	p
a	1	
b	$\frac{1}{3}$	

S	A	B	p
a	b	1	
a	d	1	

$$P(w_1) = \frac{1}{3}$$



R	A
a	
b	

S	A	B
a	b	
a	d	

R	A
a	

S	A	B
a	b	
a	d	

# Tuple-Independent PDB

Assumption:

Every tuple is an independent probabilistic event

R	A	p
a	1	
b	$\frac{1}{3}$	

S	A	B	p
a	b	1	
a	d	1	

$$P(w_1) = \frac{1}{3}$$

$$P(w_2) = \frac{2}{3}$$



R	A
a	
b	

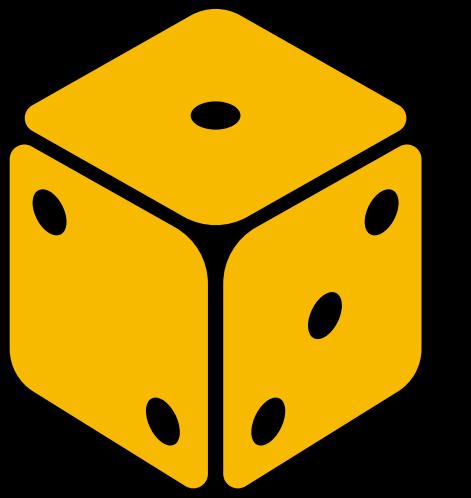
S	A	B
a	b	
a	d	

R	A
a	

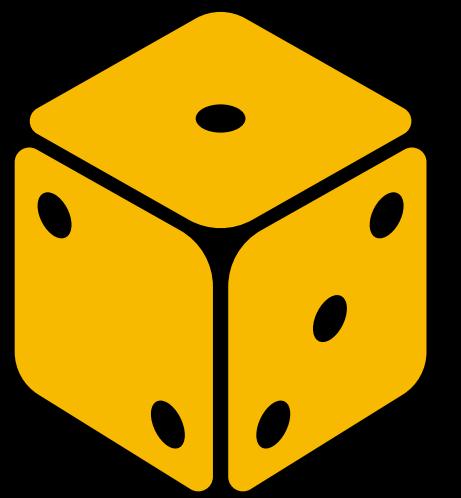
  

S	A	B
a	b	
a	d	

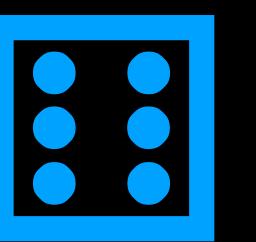
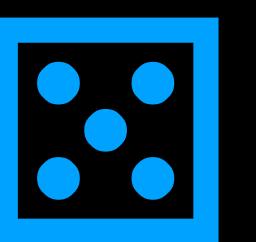
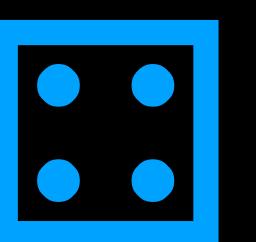
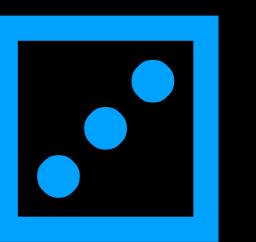
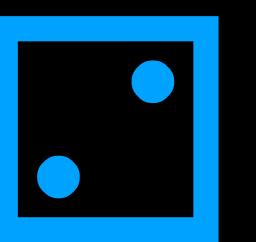
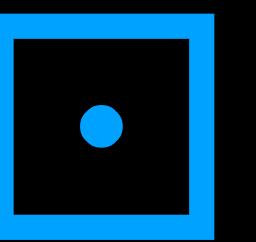
# Dice roll



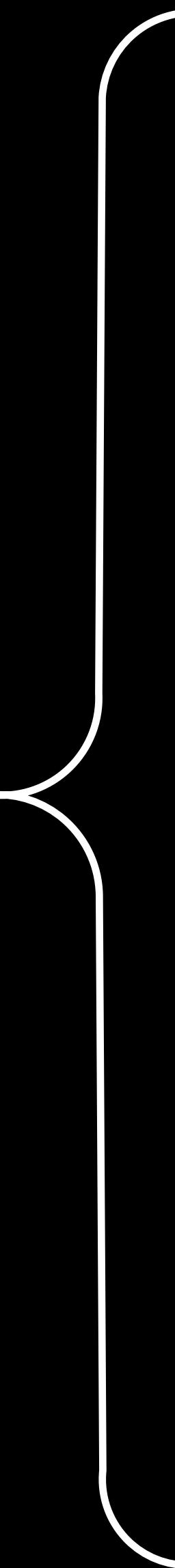
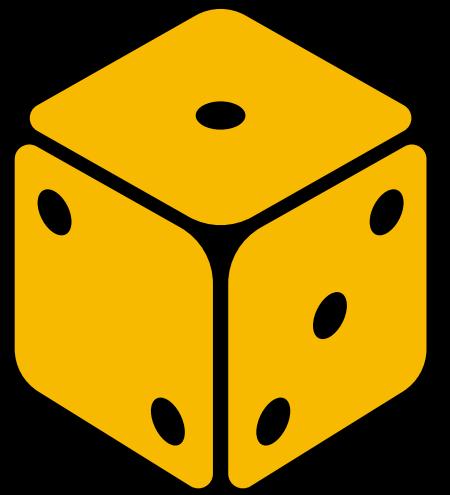
Dice roll



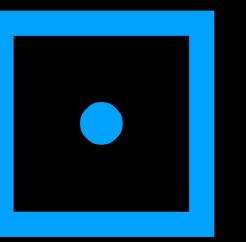
possible worlds



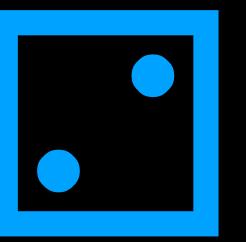
Dice roll



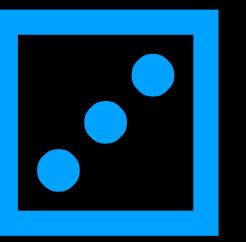
possible worlds



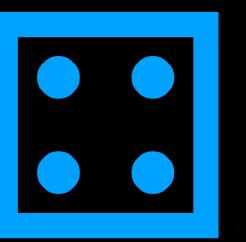
$$P = \frac{1}{6}$$



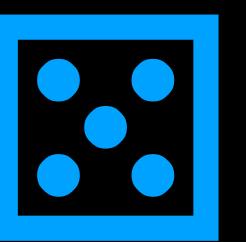
$$P = \frac{1}{6}$$



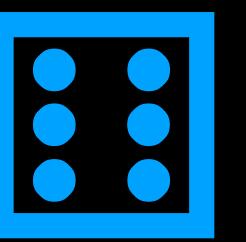
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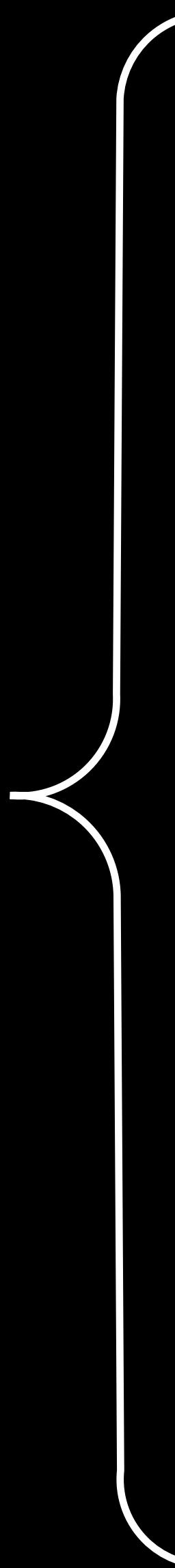
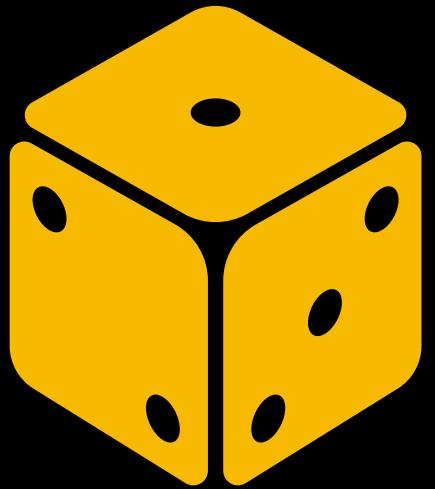


$$P = \frac{1}{6}$$

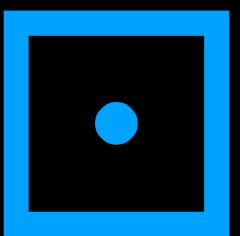


$$P = \frac{1}{6}$$

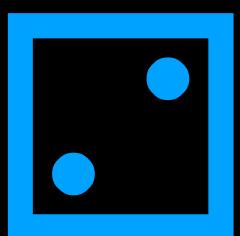
Dice roll



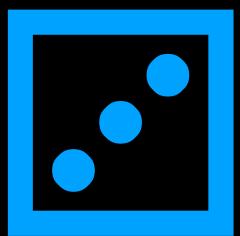
possible worlds



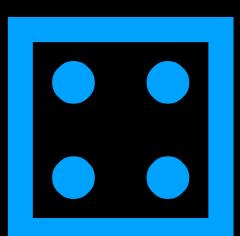
$$P = \frac{1}{6}$$



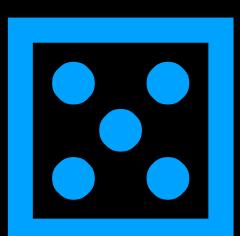
$$P = \frac{1}{6}$$



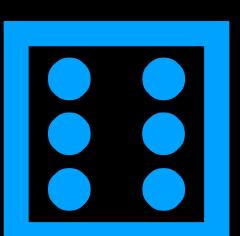
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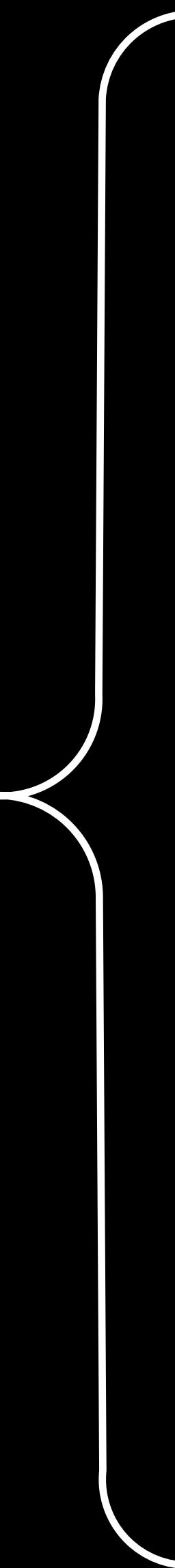
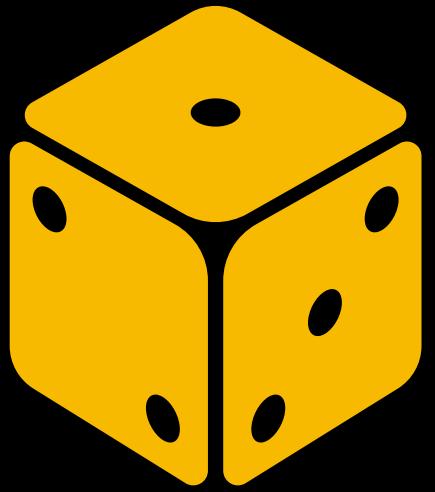


$$P = \frac{1}{6}$$

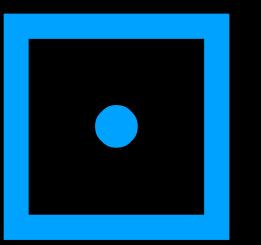
query evaluation

“Does the dice show a prime?”

# Dice roll

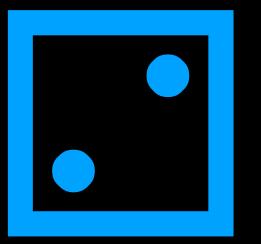


possible worlds



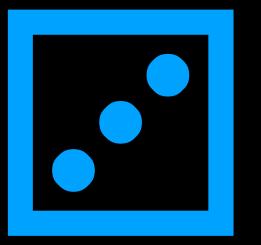
$$P = \frac{1}{6}$$

✗



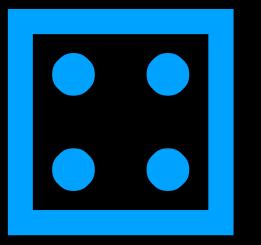
$$P = \frac{1}{6}$$

✓



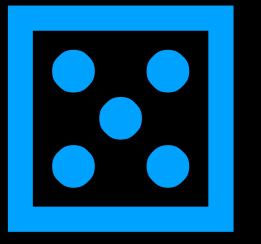
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✓



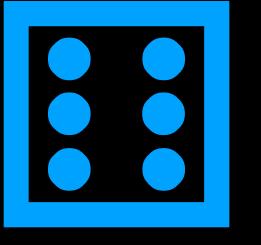
$$P = \frac{1}{6}$$

✗



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✓



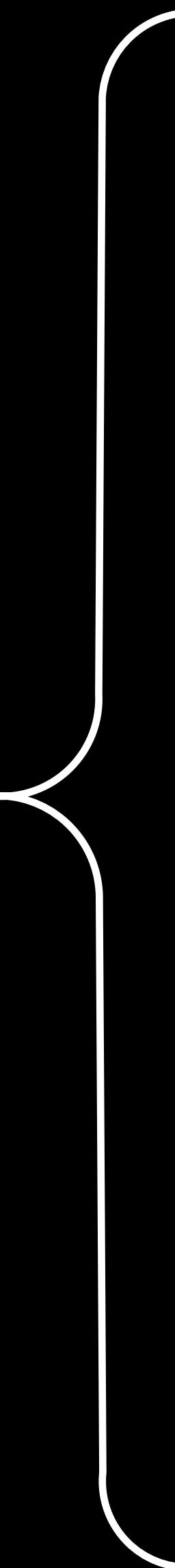
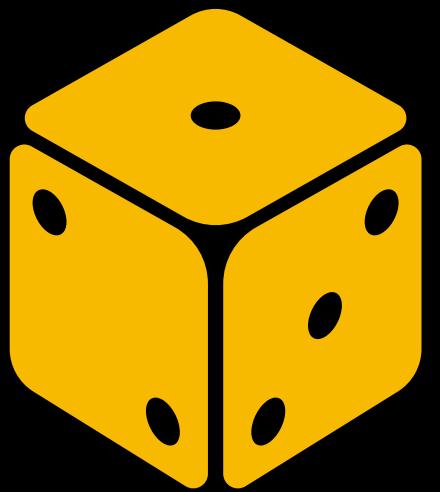
$$P = \frac{1}{6}$$

✗

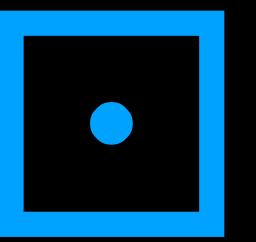
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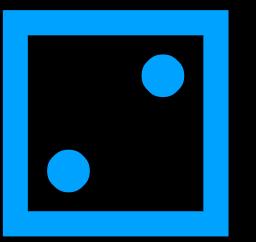


possible worlds



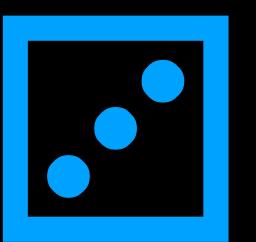
$$P = \frac{1}{6}$$

✗



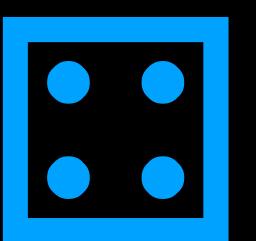
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✓



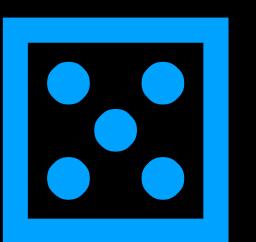
$$P = \frac{1}{6}$$

✓



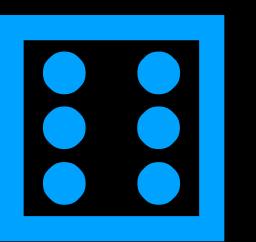
$$P = \frac{1}{6}$$

✗



$$P = \frac{1}{6}$$

✓



$$P = \frac{1}{6}$$

✗

query evaluation  
“Does the dice show a prime?”

$$\begin{aligned} P(\text{ prime }) &= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \\ &= \frac{1}{2} \end{aligned}$$

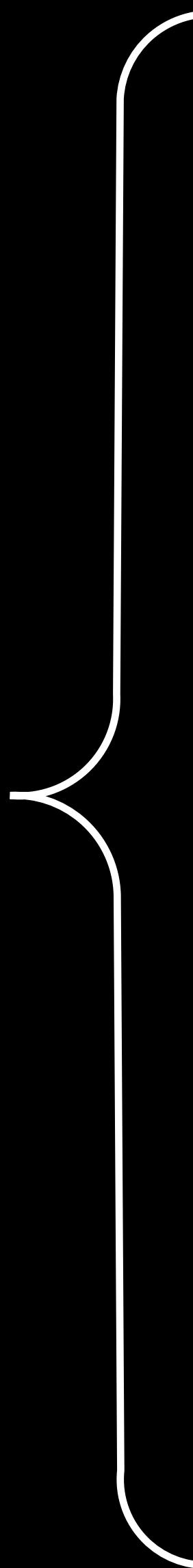
# tuple-independent PDB

R	A	p
a		$\frac{1}{2}$
b		$\frac{1}{3}$

# tuple-independent PDB

possible worlds

R	A	p
a		$\frac{1}{2}$
b		$\frac{1}{3}$



# tuple-independent PDB

possible worlds

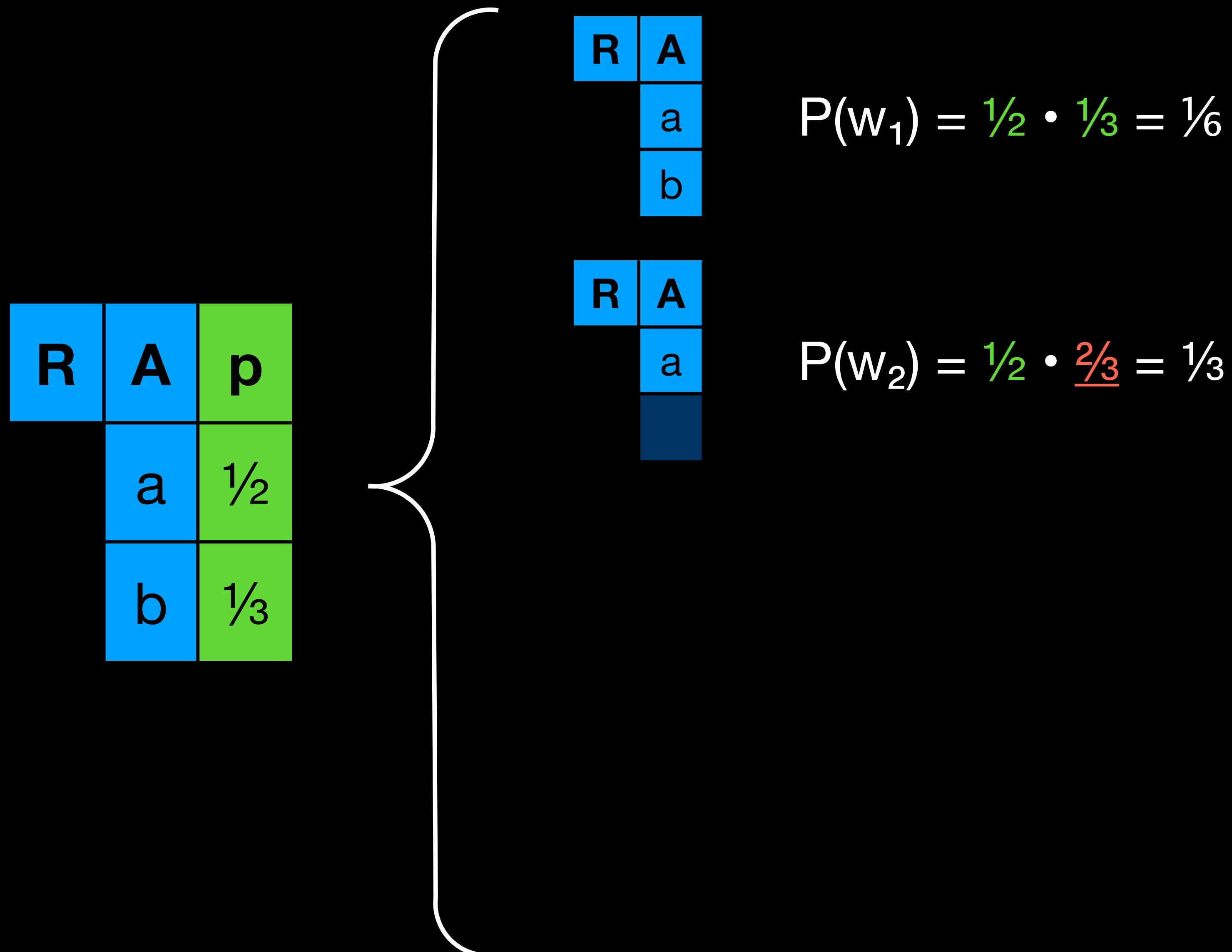
R	A	p
a		$\frac{1}{2}$
b		$\frac{1}{3}$

R	A
a	
b	

$$P(w_1) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

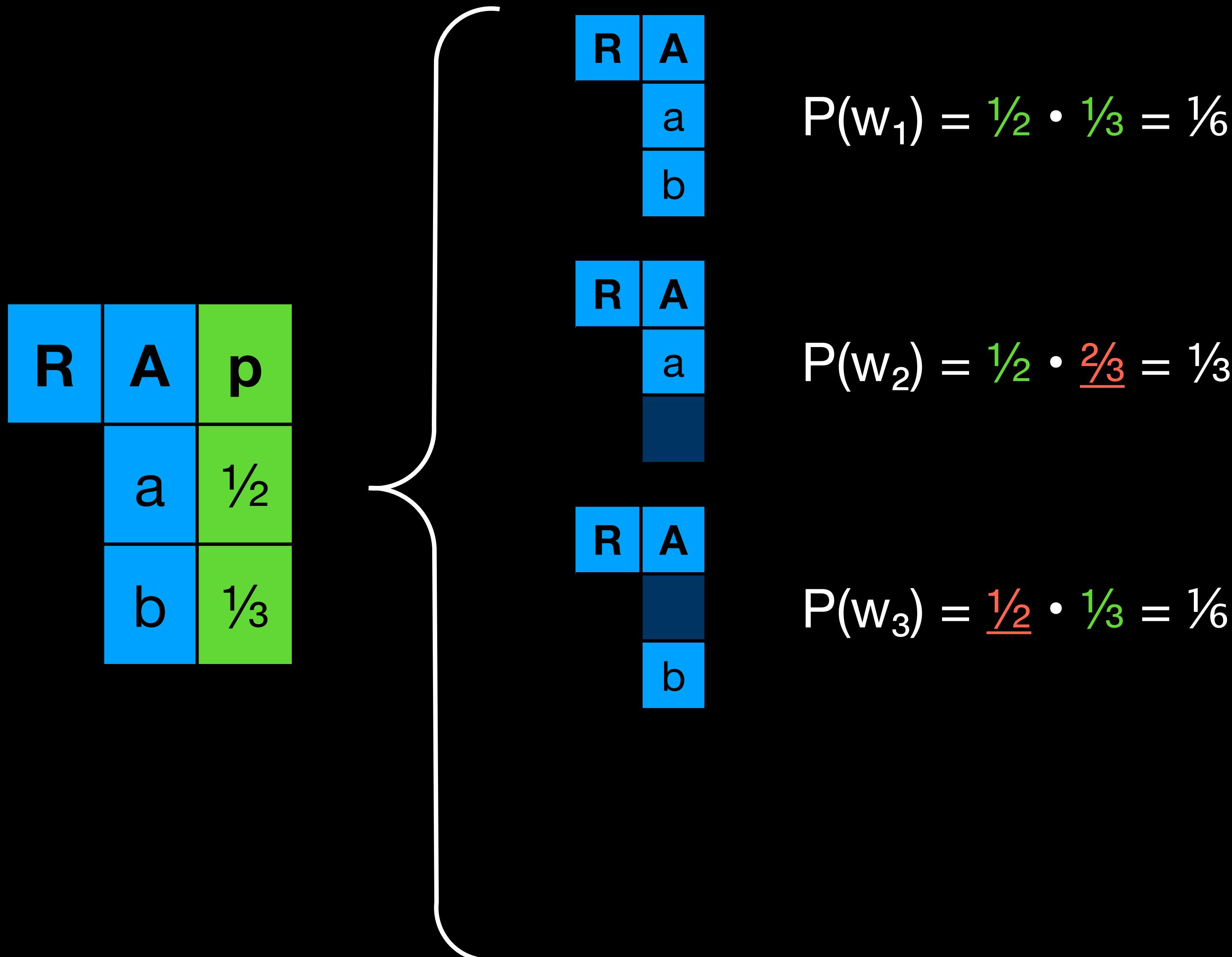
# tuple-independent PDB

possible worlds



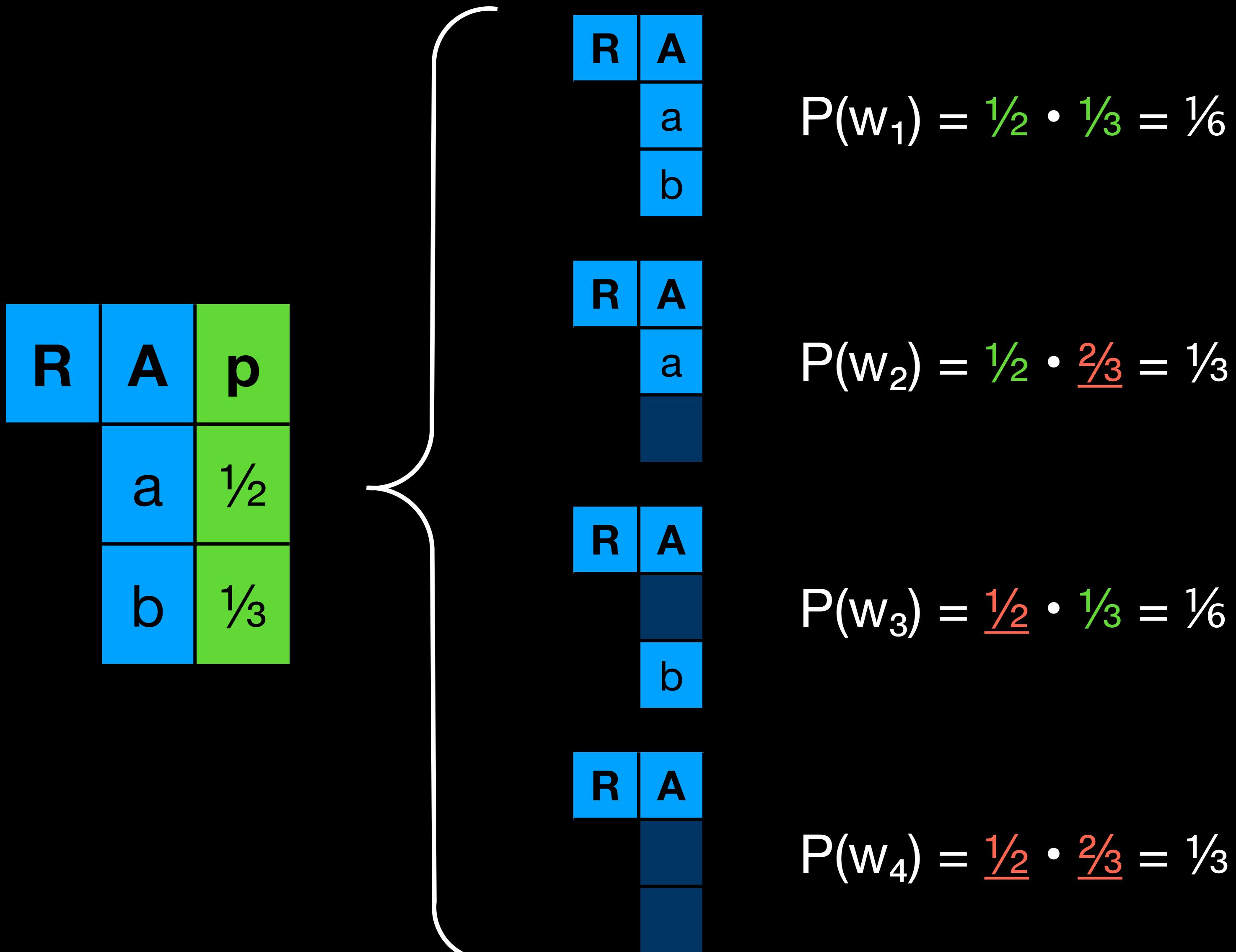
# tuple-independent PDB

possible worlds



# tuple-independent PDB

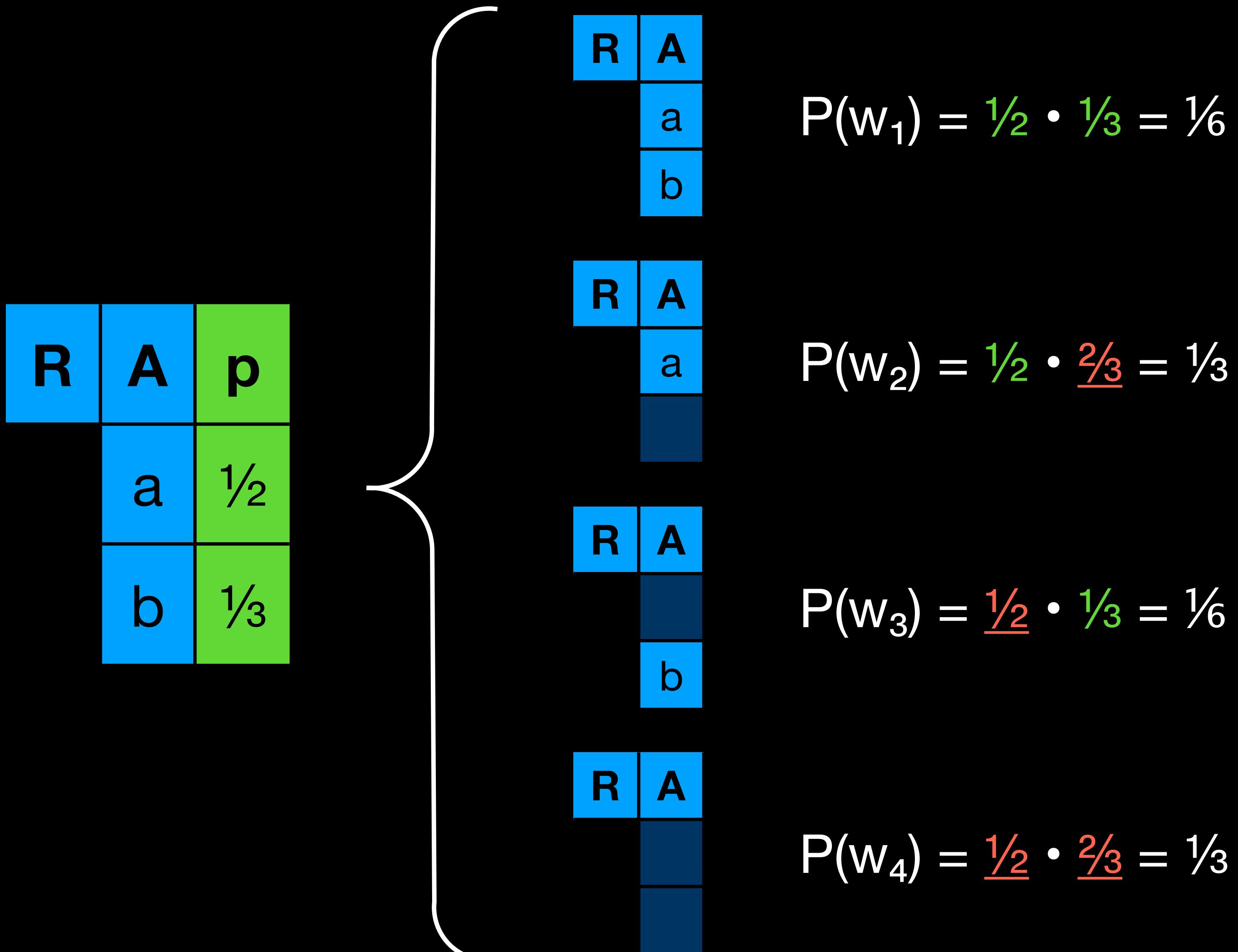
possible worlds



# tuple-independent PDB

possible worlds

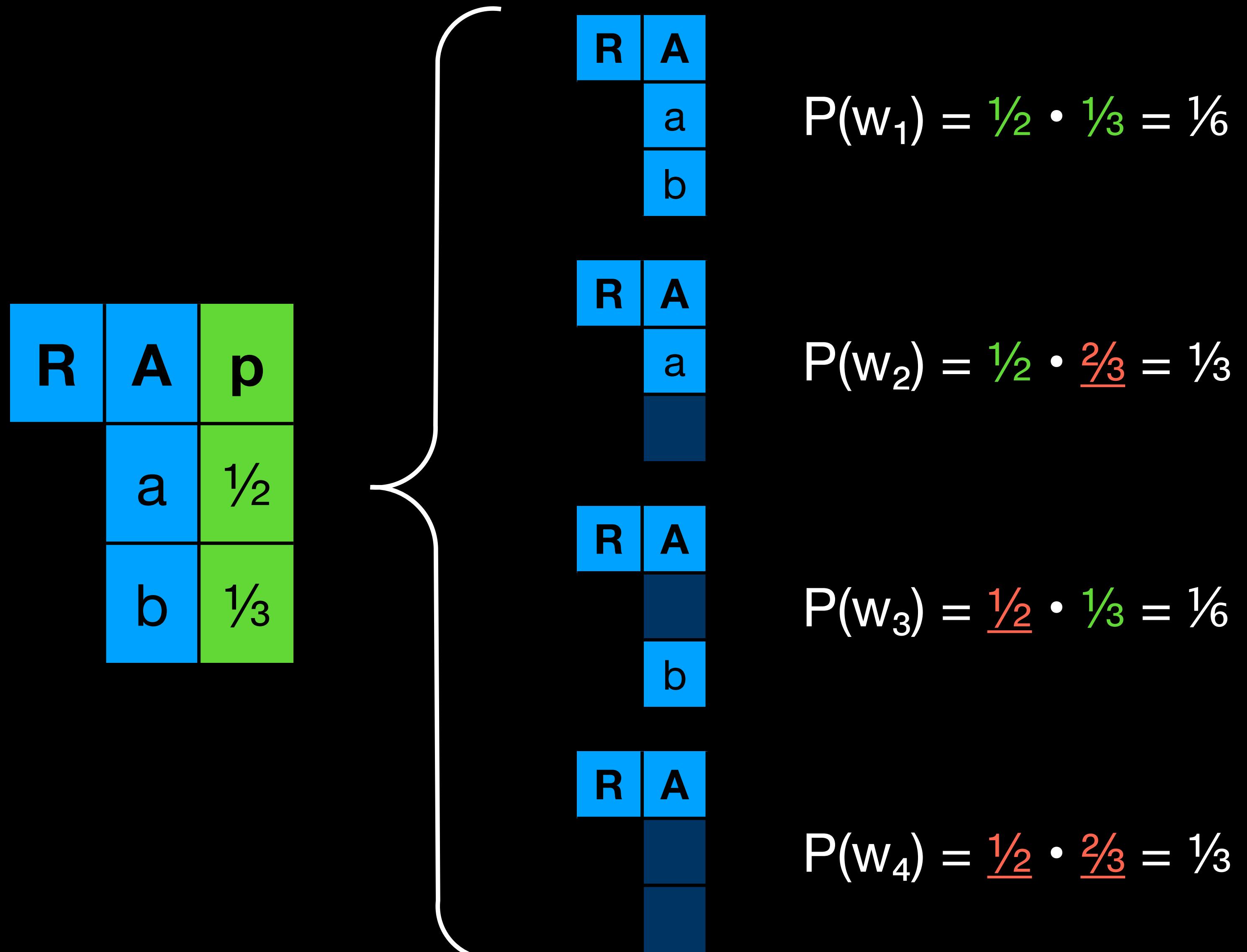
query evaluation  
 $\varphi = \exists x Rx$



# tuple-independent PDB

possible worlds

query evaluation  
 $\varphi = \exists x Rx$

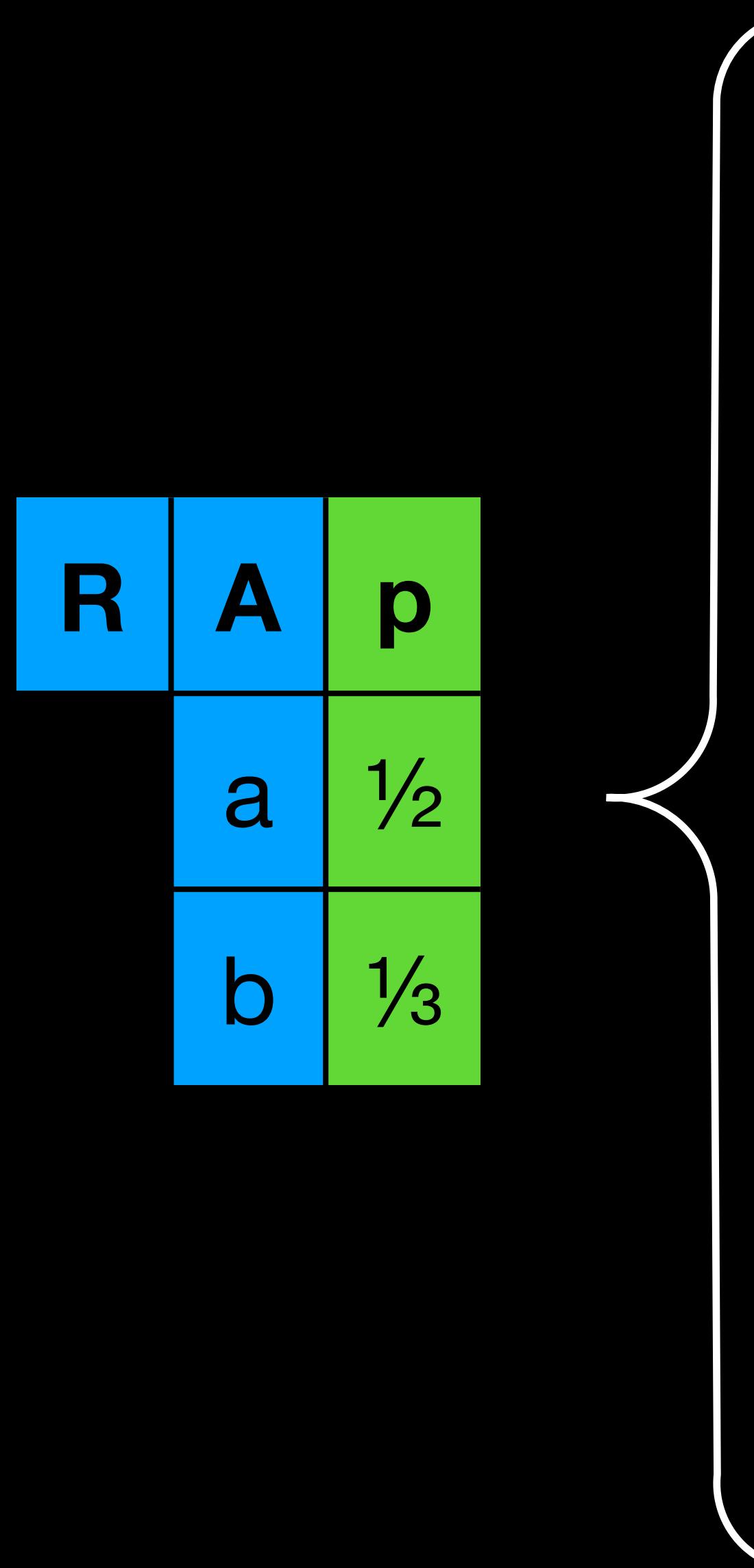


$$\varphi(w_1) = \checkmark$$

# tuple-independent PDB

possible worlds

query evaluation  
 $\varphi = \exists x Rx$



R	A
a	
b	

$$P(w_1) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$\varphi(w_1) = \checkmark$$

R	A
a	

$$P(w_2) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

$$\varphi(w_2) = \checkmark$$

R	A
b	

$$P(w_3) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

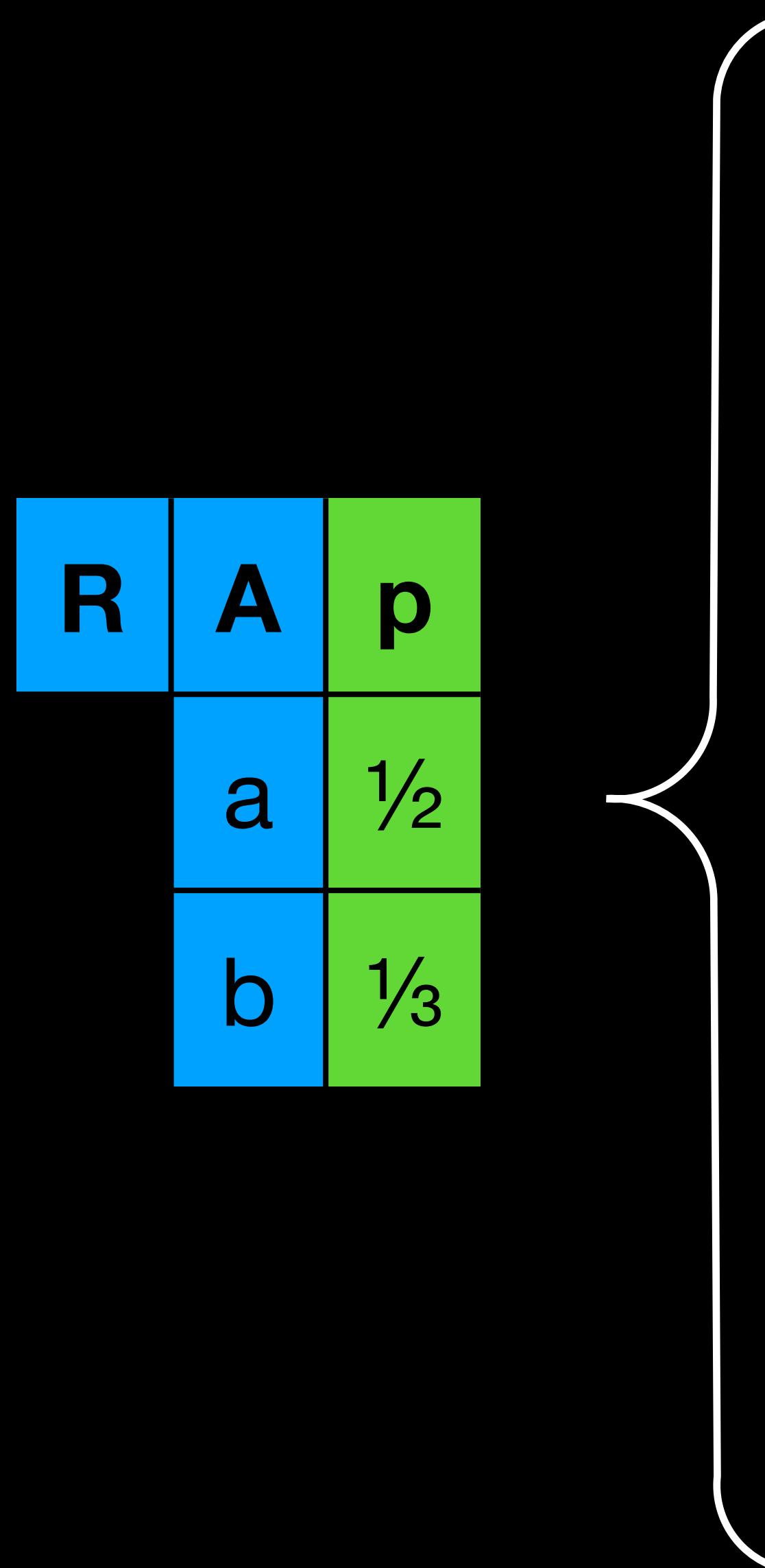
R	A

$$P(w_4) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

# tuple-independent PDB

possible worlds

query evaluation  
 $\varphi = \exists x Rx$



R	A
a	a
b	

$$P(w_1) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$\varphi(w_1) = \checkmark$$

R	A
a	a
	b

$$P(w_2) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

$$\varphi(w_2) = \checkmark$$

R	A
	a
b	

$$P(w_3) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$\varphi(w_3) = \checkmark$$

R	A
	a
b	

$$P(w_4) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

# tuple-independent PDB

possible worlds

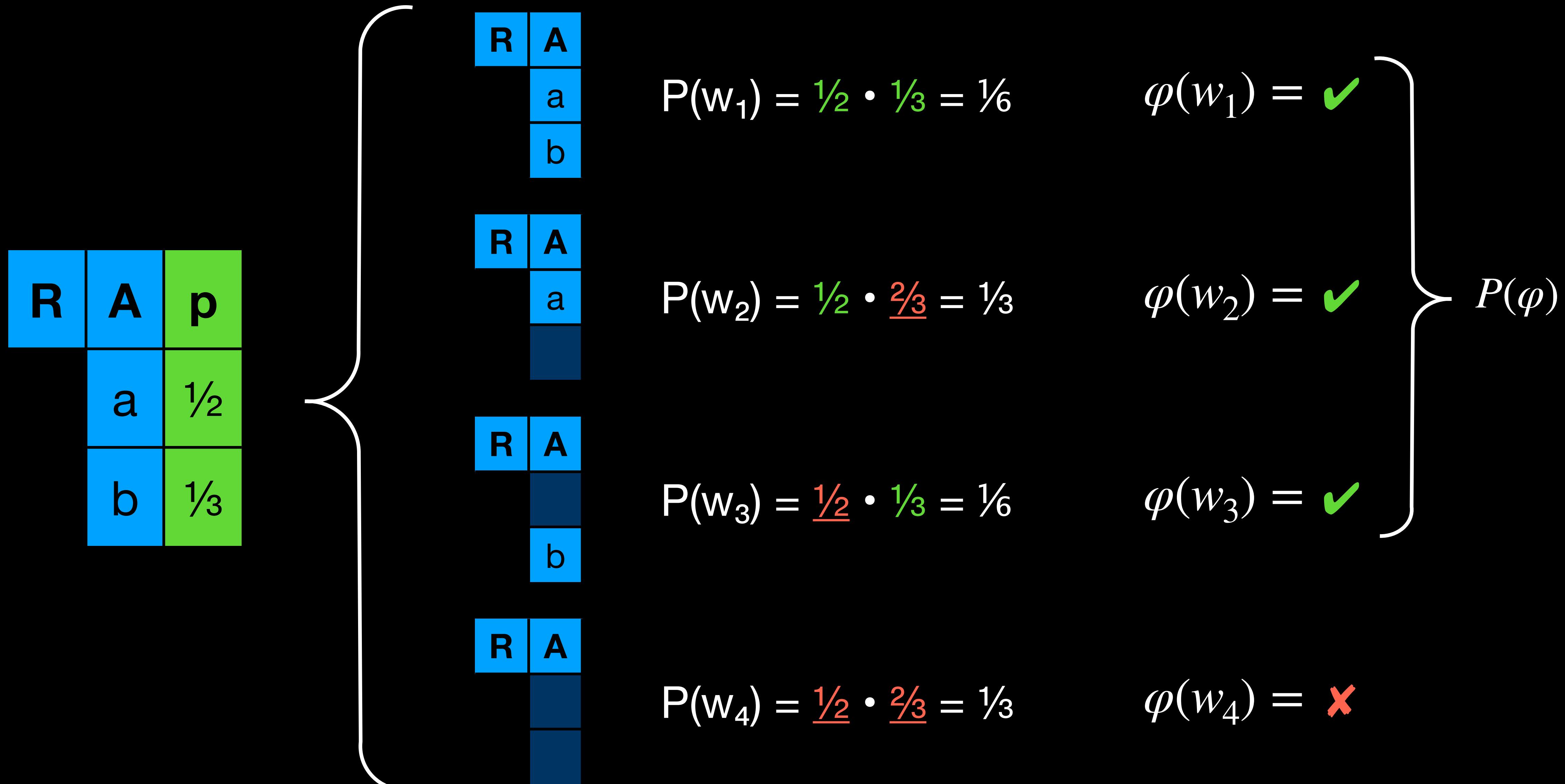
query evaluation  
 $\varphi = \exists x Rx$

tuple-independent PDB			possible worlds	query evaluation															
<table border="1"><tr><td>R</td><td>A</td><td>p</td></tr><tr><td>a</td><td></td><td><math>\frac{1}{2}</math></td></tr><tr><td>b</td><td></td><td><math>\frac{1}{3}</math></td></tr></table>	R	A	p	a		$\frac{1}{2}$	b		$\frac{1}{3}$			<table border="1"><tr><td>R</td><td>A</td></tr><tr><td>a</td><td></td></tr><tr><td>b</td><td></td></tr></table>	R	A	a		b		$P(w_1) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$
R	A	p																	
a		$\frac{1}{2}$																	
b		$\frac{1}{3}$																	
R	A																		
a																			
b																			
			<table border="1"><tr><td>R</td><td>A</td></tr><tr><td>a</td><td></td></tr><tr><td></td><td></td></tr></table>	R	A	a				$P(w_2) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$									
R	A																		
a																			
			<table border="1"><tr><td>R</td><td>A</td></tr><tr><td></td><td></td></tr><tr><td>b</td><td></td></tr></table>	R	A			b		$P(w_3) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$									
R	A																		
b																			
			<table border="1"><tr><td>R</td><td>A</td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>	R	A					$P(w_4) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$									
R	A																		

# tuple-independent PDB

possible worlds

query evaluation  
 $\varphi = \exists x Rx$



# tuple-independent PDB

possible worlds

query evaluation  
 $\varphi = \exists x Rx$

<b>R</b>	<b>A</b>	<b>p</b>			
a		$\frac{1}{2}$	$P(w_1) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$	$\varphi(w_1) = \checkmark$	$P(\varphi) = \frac{1}{6} + \frac{1}{3} + \frac{1}{6} = \frac{2}{3}$
b		$\frac{1}{3}$	$P(w_2) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$	$\varphi(w_2) = \checkmark$	
			$P(w_3) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$	$\varphi(w_3) = \checkmark$	
			$P(w_4) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$	$\varphi(w_4) = \times$	

# 1 Probabilistic Databases 2 under Updates

- 3 Boolean Query Evaluation
- 4 and Ranked Enumeration

Christoph Berkholz, Maximilian Merz

Talk at AIMoTh 2022, based on the PODS 2021 paper



# Probabilistic Databases

②

## under Updates

③

Boolean Query Evaluation  
and Ranked Enumeration

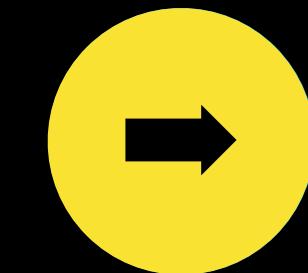
④

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# Probabilistic Databases



## under Updates

3

Boolean Query Evaluation  
and Ranked Enumeration

4

Christoph Berkholz, Maximilian Merz

Talk at AIMoTh 2022, based on the PODS 2021 paper

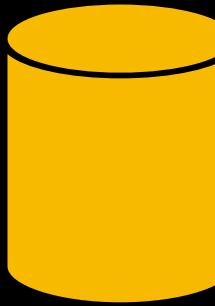
# Updates to the Database



# Updates to the Database

init

- fix db schema
- fix finite domain
- fix FO query



time

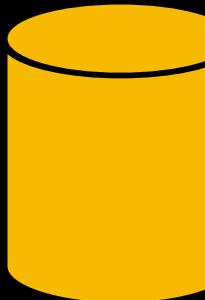
# Updates to the Database

init

- fix db schema
- fix finite domain
- fix FO query

R | A | p

S | A | B | p



time

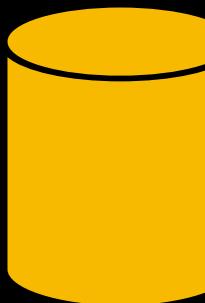
# Updates to the Database

init

- fix db schema
- fix finite domain
- fix FO query

R	A	p
a	$\frac{3}{4}$	
b	$\frac{1}{3}$	

S	A	B	p
a		b	$\frac{1}{8}$



insert

insert

insert

time

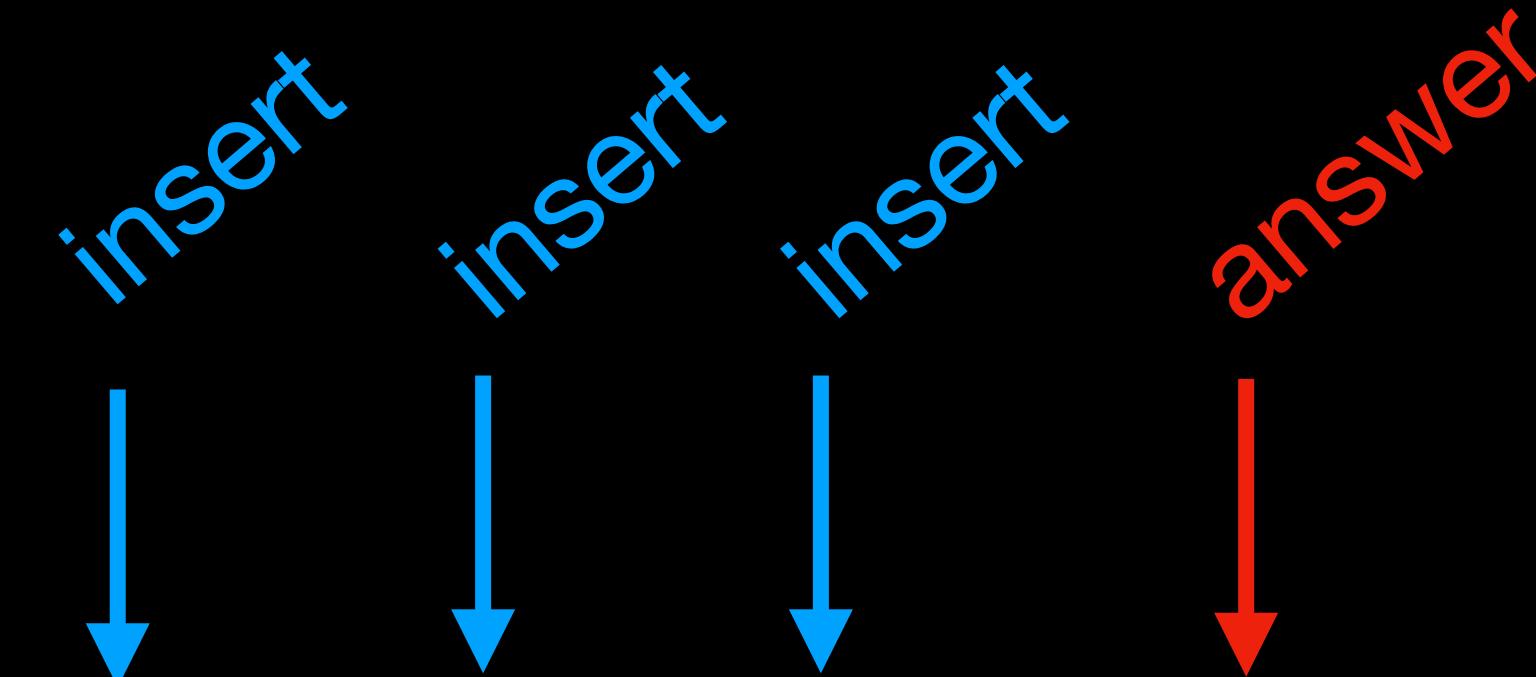
# Updates to the Database

init

- fix db schema
- fix finite domain
- fix FO query

R	A	p
a	$\frac{3}{4}$	
b	$\frac{1}{3}$	

S	A	B	p
a	b		$\frac{1}{8}$



# Updates to the Database

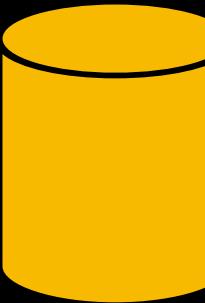
init

- fix db schema
- fix finite domain
- fix FO query

R	A	p
---	---	---

b	$\frac{1}{3}$
---	---------------

S	A	B	p
a	b		$\frac{1}{8}$
a	d		1



insert

insert

insert

answer

insert

delete

answer

...

time

# Updates to the Database

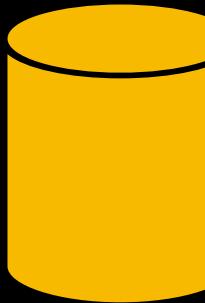
init

- fix db schema
- fix finite domain
- fix FO query

R	A	p
---	---	---

b	$\frac{1}{3}$
---	---------------

S	A	B	p
a	b		$\frac{1}{8}$
a	d		1



insert

insert

insert

answer

insert

delete

answer

...

time

update time

# Updates to the Database

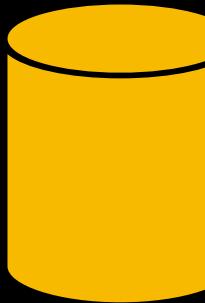
init

- fix db schema
- fix finite domain
- fix FO query

R	A	p
---	---	---

b	$\frac{1}{3}$
---	---------------

S	A	B	p
a	b		$\frac{1}{8}$
a	d		1



insert

insert

insert

answer

insert

delete

answer

...

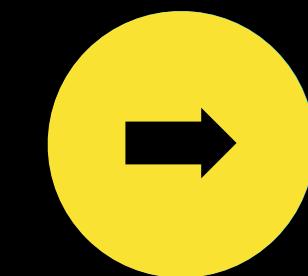
time

update time

answer time



# Probabilistic Databases



## under Updates

- 3 Boolean Query Evaluation
- 4 and Ranked Enumeration

Christoph Berkholz, Maximilian Merz

Talk at AIMoTh 2022, based on the PODS 2021 paper



# Probabilistic Databases



## under Updates

3

Boolean Query Evaluation  
and Ranked Enumeration

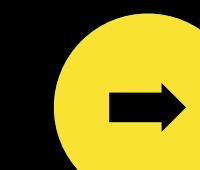
4

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# Probabilistic Databases under Updates



Boolean Query Evaluation  
and Ranked Enumeration

4

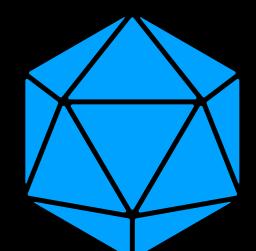
Christoph Berkholz, Maximilian Merz

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Boolean Query Evaluation

# Dalvi & Suciu Algorithm

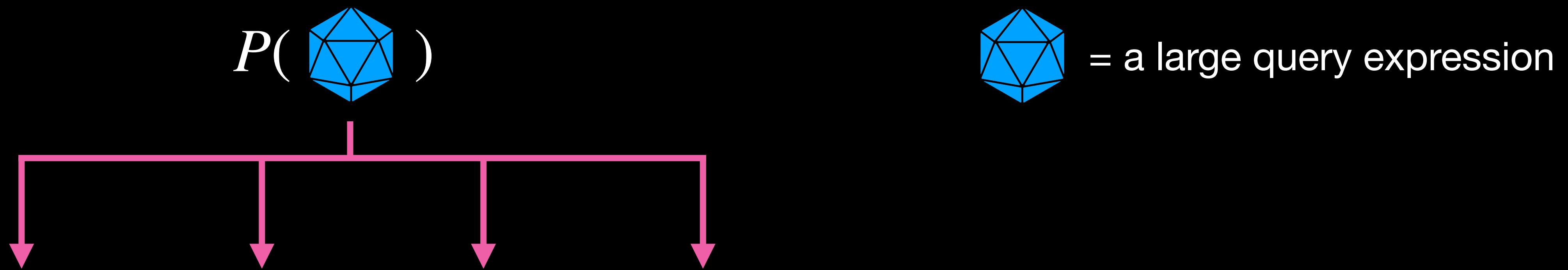
$$P( \text{ } \text{ } \text{ } )$$



= a large query expression

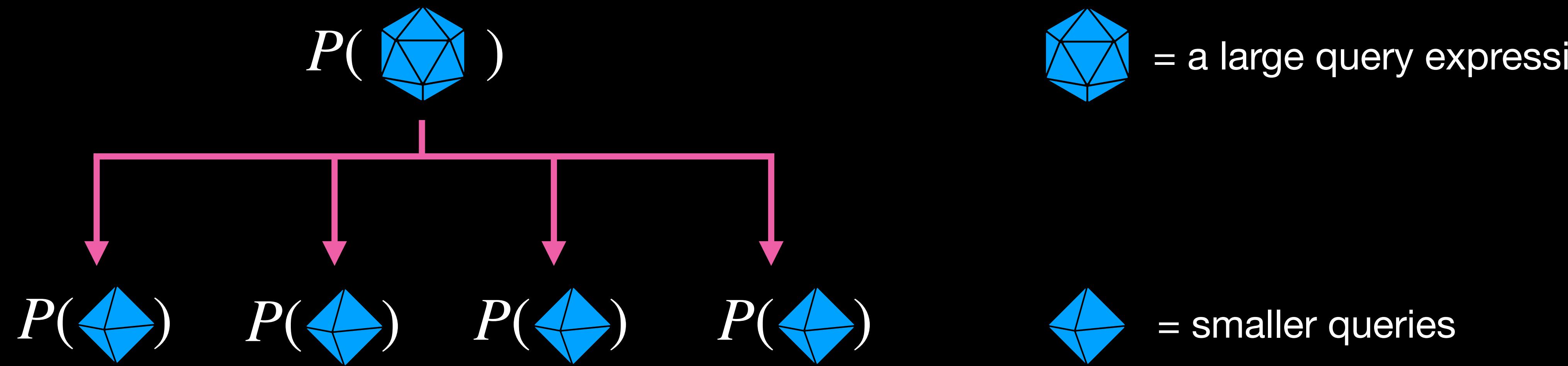
Boolean Query Evaluation

# Dalvi & Suciu Algorithm



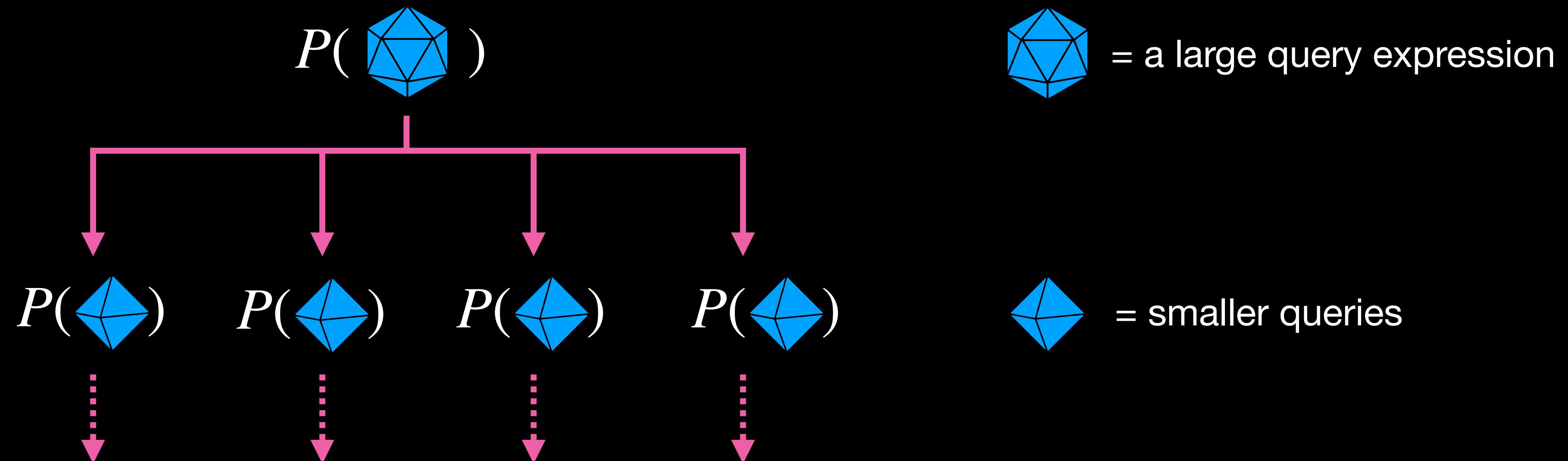
# Boolean Query Evaluation

## Dalvi & Suciu Algorithm



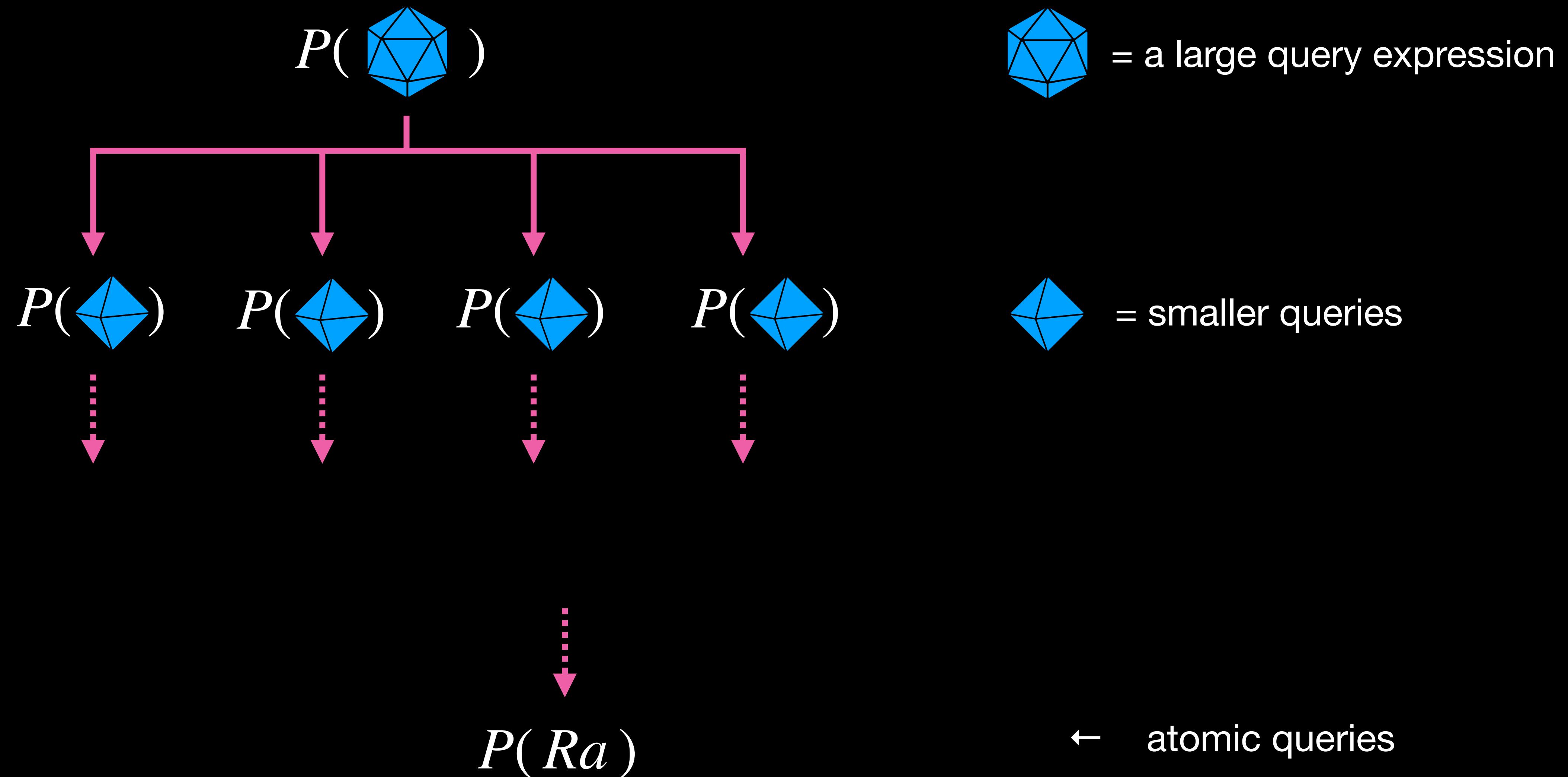
# Boolean Query Evaluation

## Dalvi & Suciu Algorithm



# Boolean Query Evaluation

## Dalvi & Suciu Algorithm



Boolean Query Evaluation

# Dalvi & Suciu Algorithm

Boolean Query Evaluation

# Dalvi & Suciu Algorithm

Syntactical Independence 1

$$P( \exists x Rx \wedge \exists y Sy )$$

Boolean Query Evaluation

# Dalvi & Suciu Algorithm

Syntactical Independence 1

$$P(\exists x Rx \wedge \exists y Sy)$$

## Boolean Query Evaluation

# Dalvi & Suciu Algorithm

### Syntactical Independence 1

$$P(\text{ } \exists x Rx \text{ } \wedge \text{ } \exists y Sy)$$

$$= P(\text{ } \exists x Rx) \cdot P(\text{ } \exists y Sy)$$

# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

Syntactical Independence 1

$$P(\text{ } \exists x Rx \text{ } \wedge \text{ } \exists y Sy)$$

Syntactical Independence 2

$$= P(\text{ } \exists x Rx) \cdot P(\text{ } \exists y Sy)$$

# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

Syntactical Independence 1

$$P(\exists x Rx \wedge \exists y Sy)$$

$$= P(\exists x Rx) \cdot P(\exists y Sy)$$

Syntactical Independence 2

$$P(\exists x Rx)$$

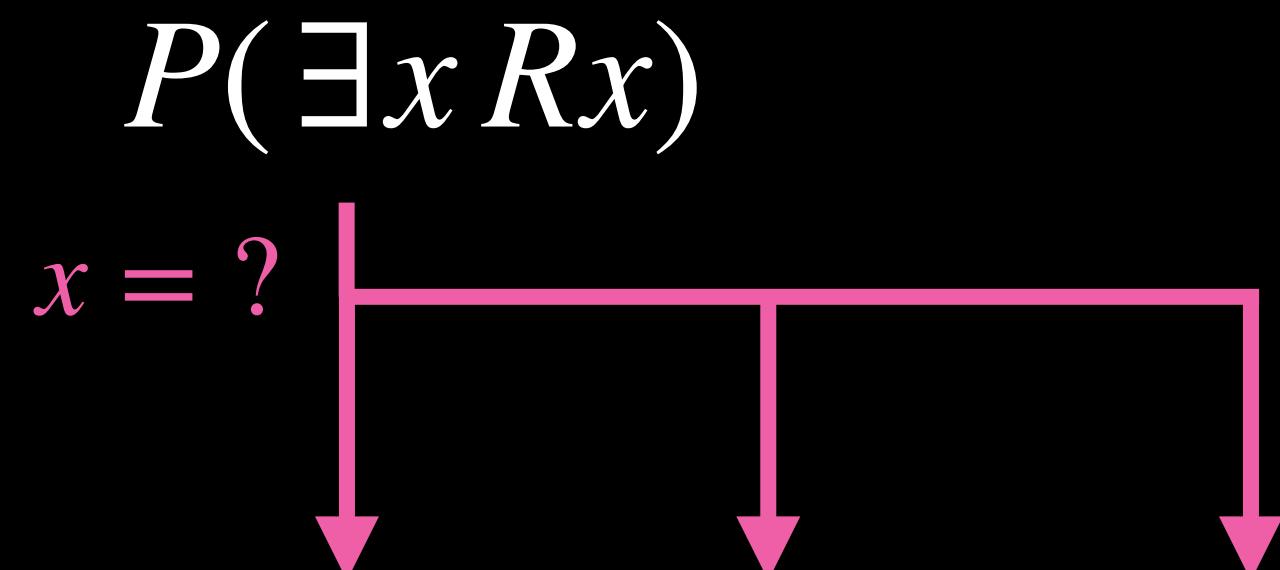
# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

Syntactical Independence 1

$$\begin{aligned} P(\exists x Rx \wedge \exists y Sy) \\ = P(\exists x Rx) \cdot P(\exists y Sy) \end{aligned}$$

Syntactical Independence 2



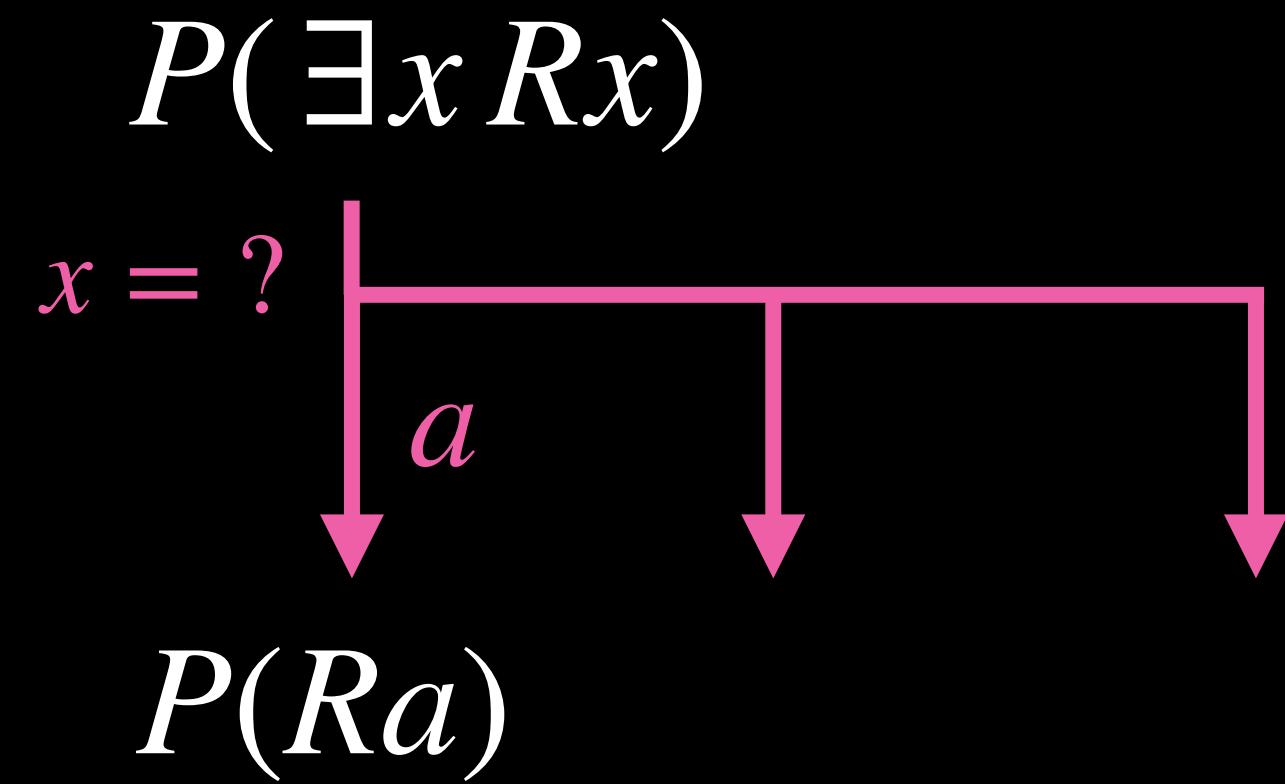
# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

### Syntactical Independence 1

$$\begin{aligned} P(\exists x Rx \wedge \exists y Sy) \\ = P(\exists x Rx) \cdot P(\exists y Sy) \end{aligned}$$

### Syntactical Independence 2



# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

### Syntactical Independence 1

$$\begin{aligned} & P(\exists x Rx \wedge \exists y Sy) \\ &= P(\exists x Rx) \cdot P(\exists y Sy) \end{aligned}$$

### Syntactical Independence 2

$$\begin{aligned} & P(\exists x Rx) \\ & x = ? \quad \begin{array}{c} \downarrow a \quad \downarrow b \quad \downarrow c \\ P(Ra) \quad P(Rb) \quad P(Rc) \end{array} \end{aligned}$$

# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

### Syntactical Independence 1

$$\begin{aligned} P(\exists x Rx \wedge \exists y Sy) \\ = P(\exists x Rx) \cdot P(\exists y Sy) \end{aligned}$$

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...if x is a separator variable!

# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

### Syntactical Independence 1

$$\begin{aligned} P(\exists x Rx \wedge \exists y Sy) \\ = P(\exists x Rx) \cdot P(\exists y Sy) \end{aligned}$$

### Syntactical Independence 2

$$\begin{aligned} P(\exists x Rx) \\ x = ? \quad \begin{array}{c} | \\ a \quad b \quad c \\ \downarrow \quad \downarrow \quad \downarrow \\ P(Ra) \quad P(Rb) \quad P(Rc) \end{array} \end{aligned}$$

...if x is a separator variable!

### Inclusion/Exclusion

# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

### Syntactical Independence 1

$$\begin{aligned} P(\exists x Rx \wedge \exists y Sy) \\ = P(\exists x Rx) \cdot P(\exists y Sy) \end{aligned}$$

### Syntactical Independence 2

$$\begin{aligned} P(\exists x Rx) \\ x = ? \quad | \\ \downarrow a \quad \downarrow b \quad \downarrow c \\ P(Ra) \quad P(Rb) \quad P(Rc) \end{aligned}$$

...if x is a separator variable!

### Inclusion/Exclusion

$$P(\varphi_1 \wedge \varphi_2)$$

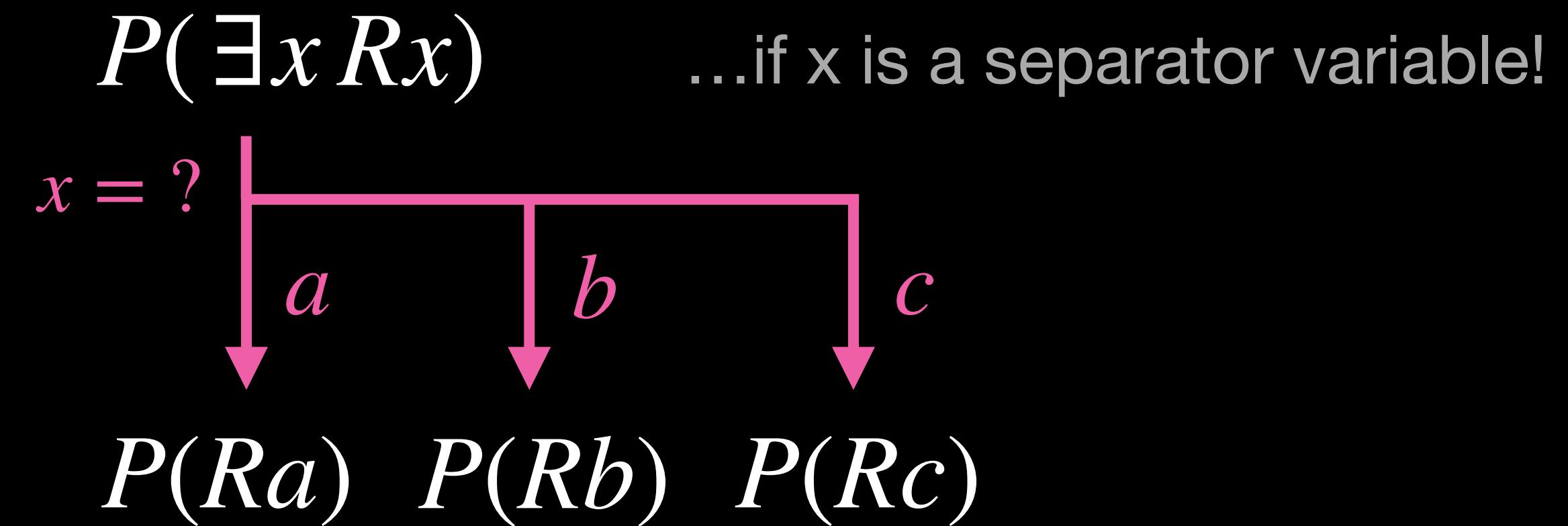
# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

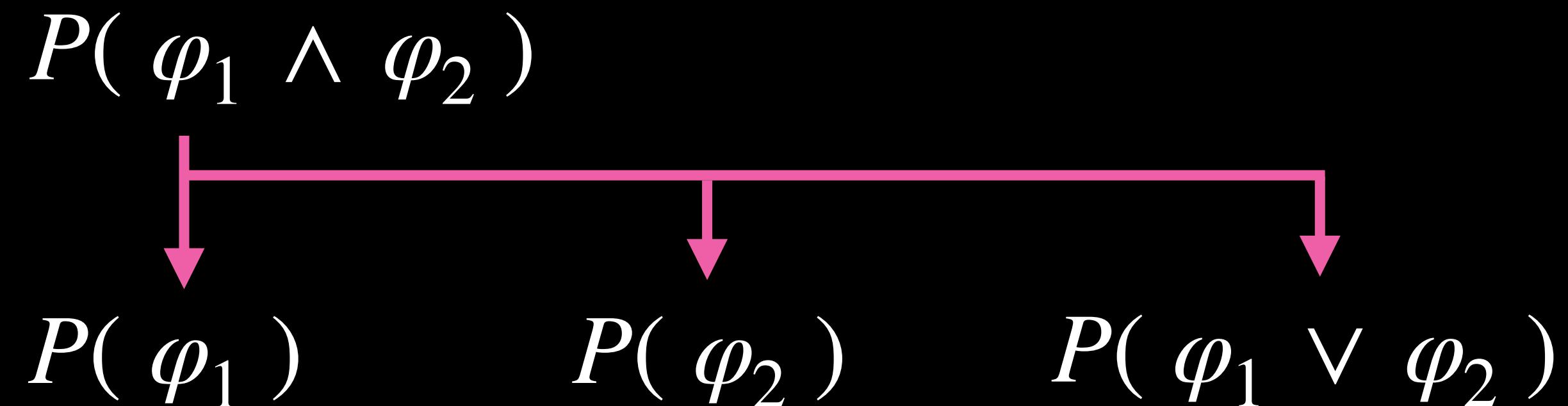
### Syntactical Independence 1

$$\begin{aligned} P(\exists x Rx \wedge \exists y Sy) \\ = P(\exists x Rx) \cdot P(\exists y Sy) \end{aligned}$$

### Syntactical Independence 2



### Inclusion/Exclusion



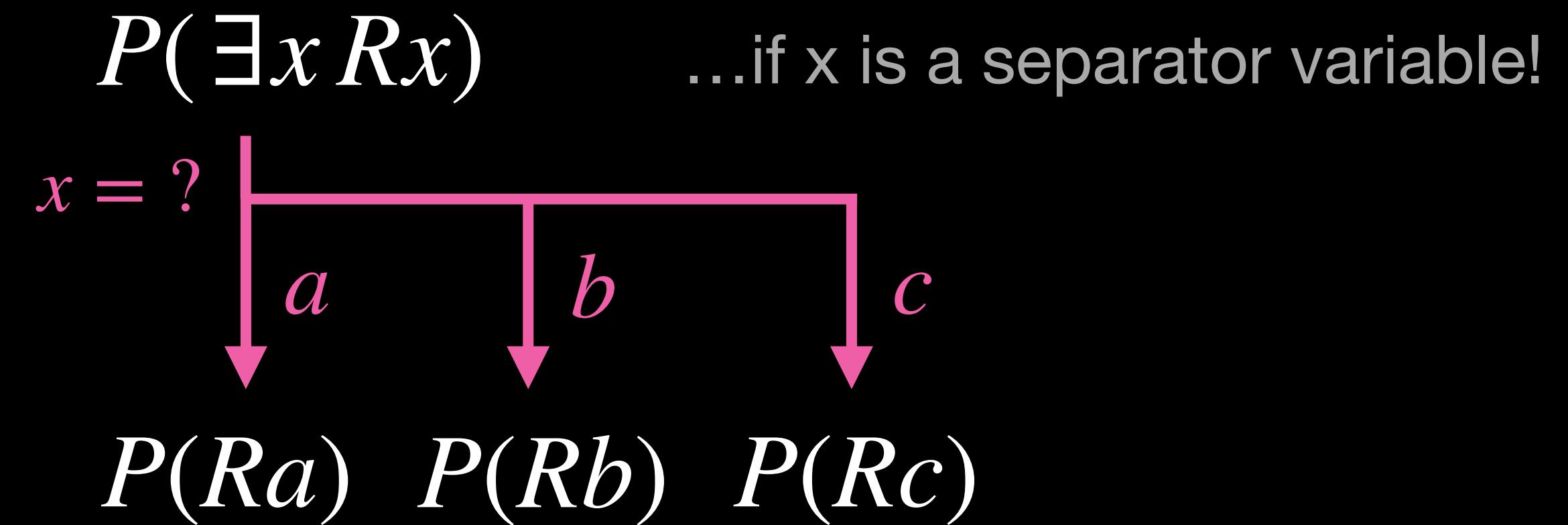
# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

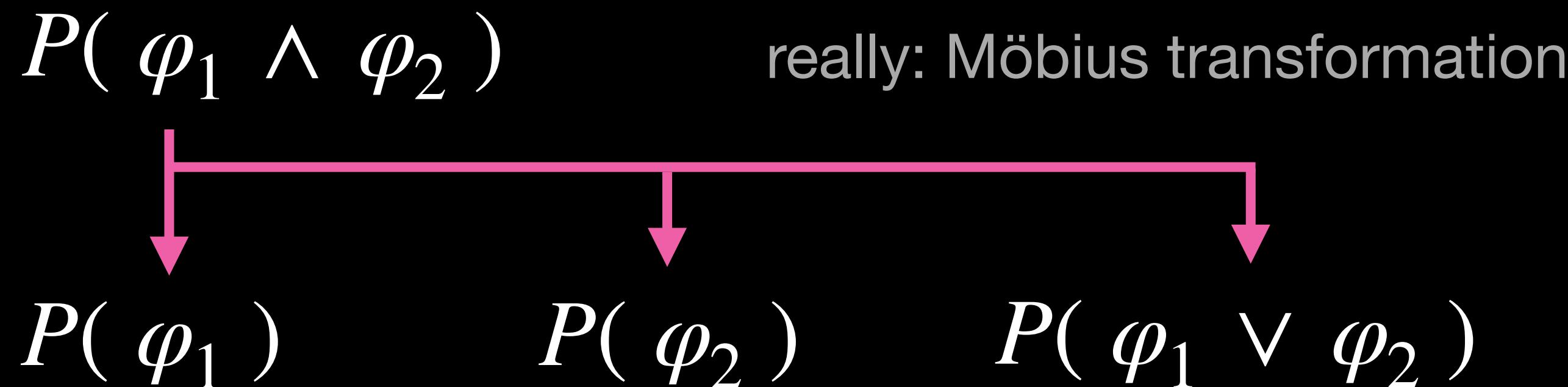
### Syntactical Independence 1

$$\begin{aligned} P(\exists x Rx \wedge \exists y Sy) \\ = P(\exists x Rx) \cdot P(\exists y Sy) \end{aligned}$$

### Syntactical Independence 2

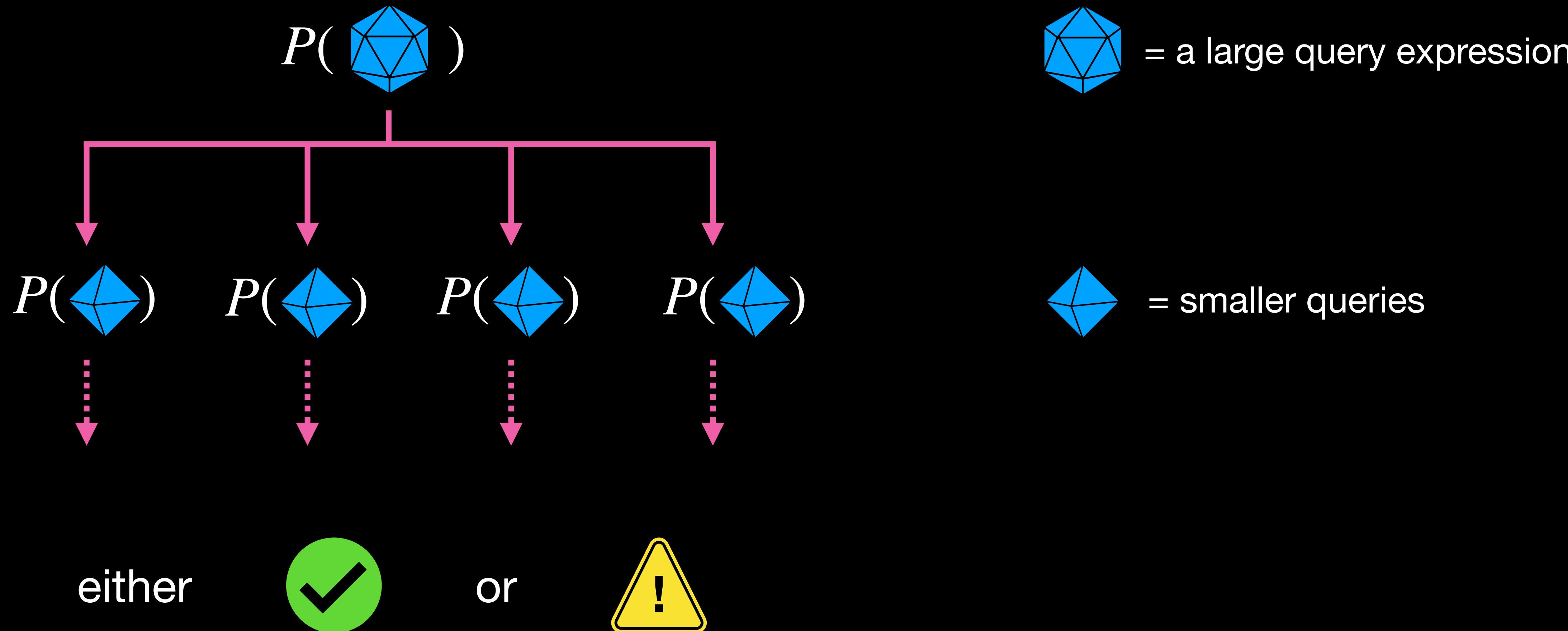


### Inclusion/Exclusion



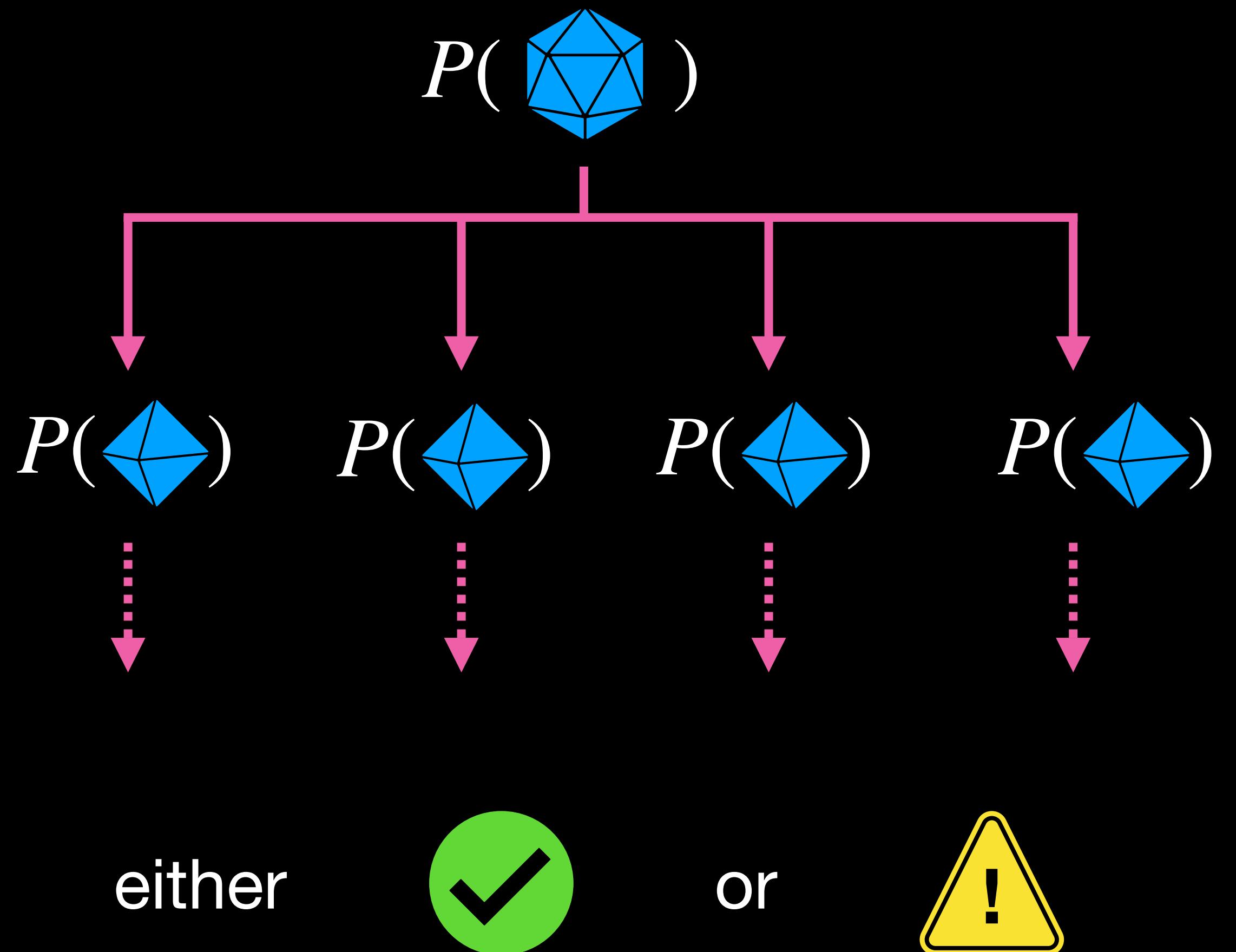
# Boolean Query Evaluation

## Dalvi & Suciu Algorithm



# Boolean Query Evaluation

# Dalvi & Suciu Algorithm



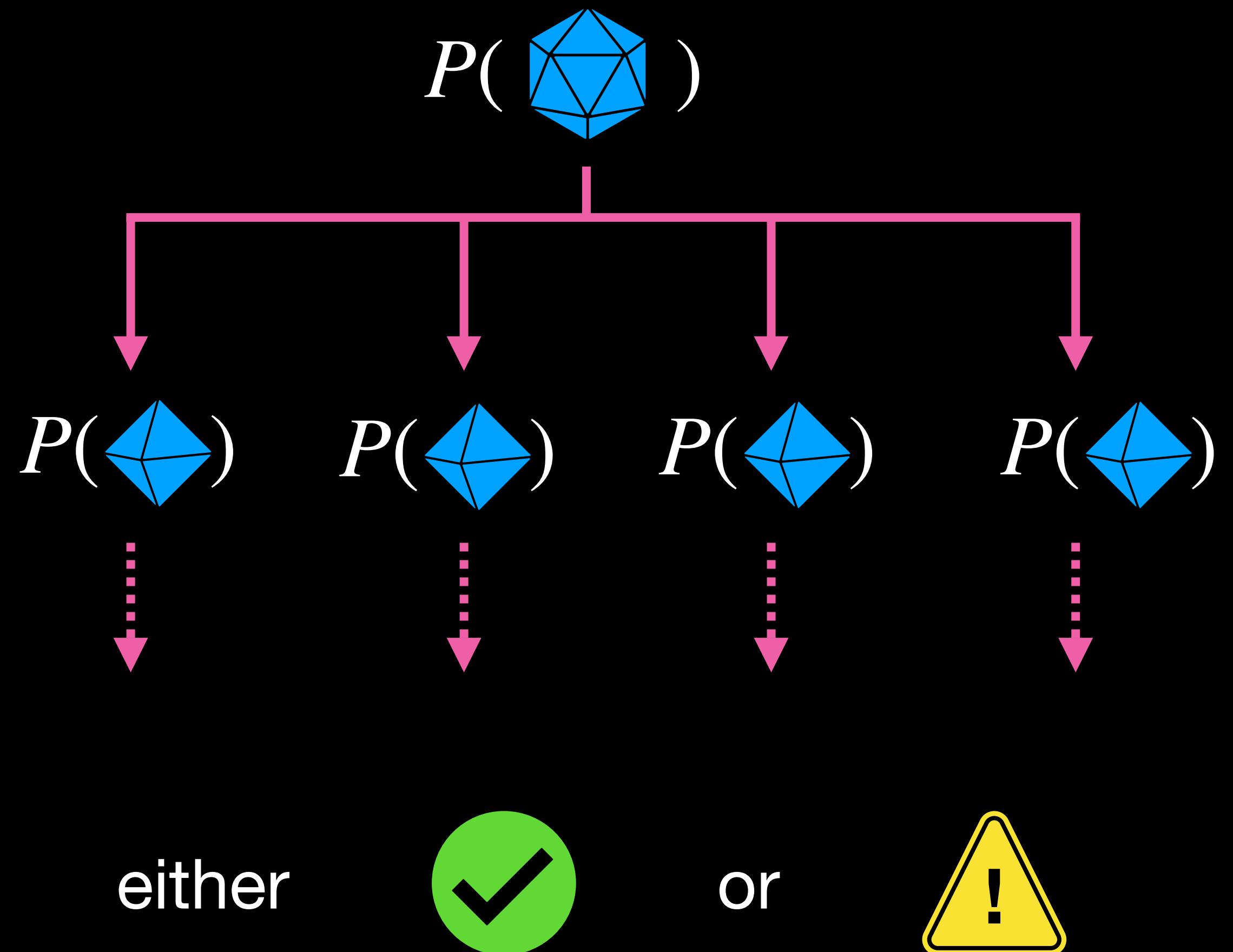
= a large query expression

= smaller queries

**Dichotomy [Dalvi & Suciu 2012]:**

# Boolean Query Evaluation

# Dalvi & Suciu Algorithm



= a large query expression

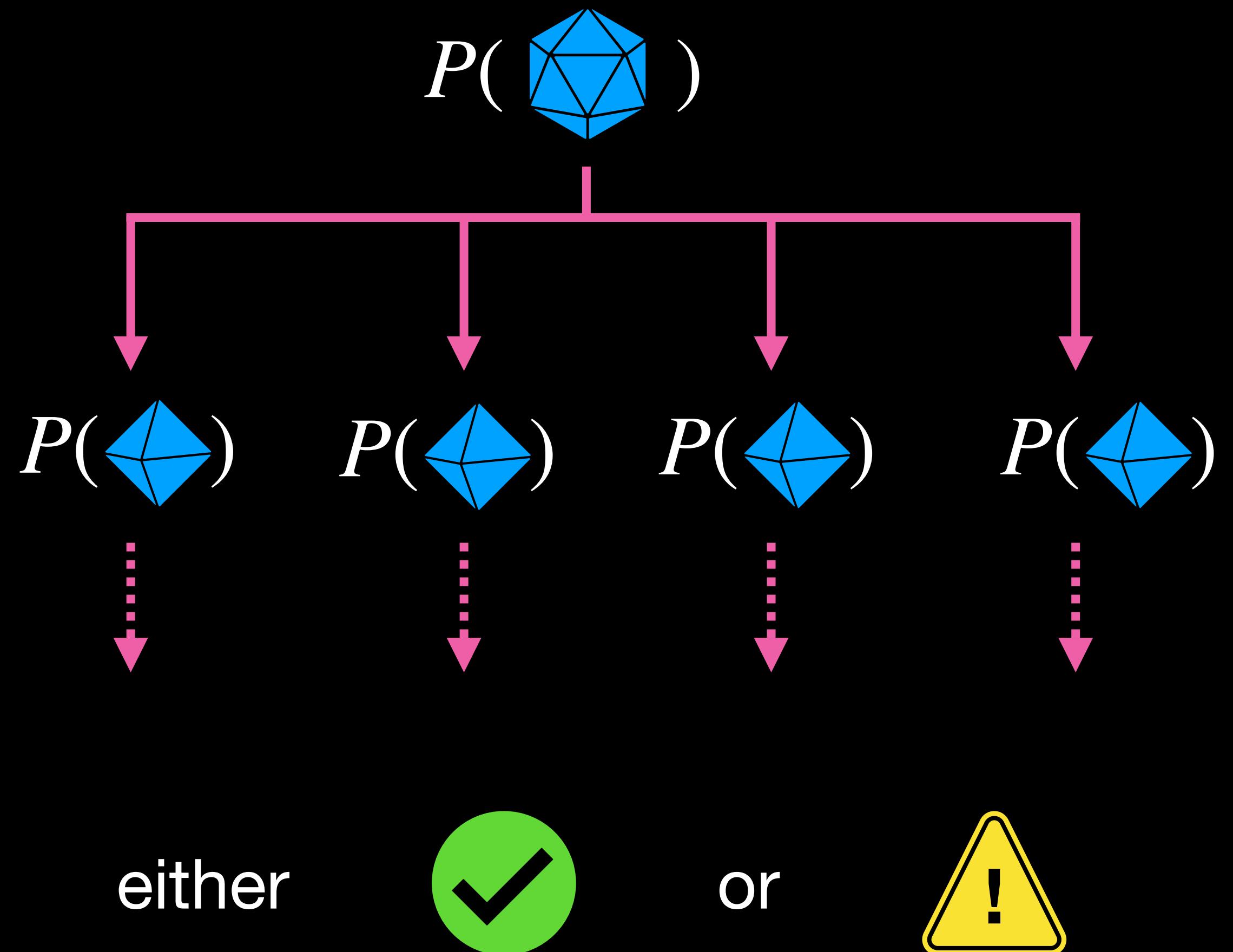
= smaller queries

**Dichotomy [Dalvi & Suciu 2012]:**

then  $P(\text{large query expression})$  is in PTIME

# Boolean Query Evaluation

## Dalvi & Suciu Algorithm



= a large query expression

= smaller queries

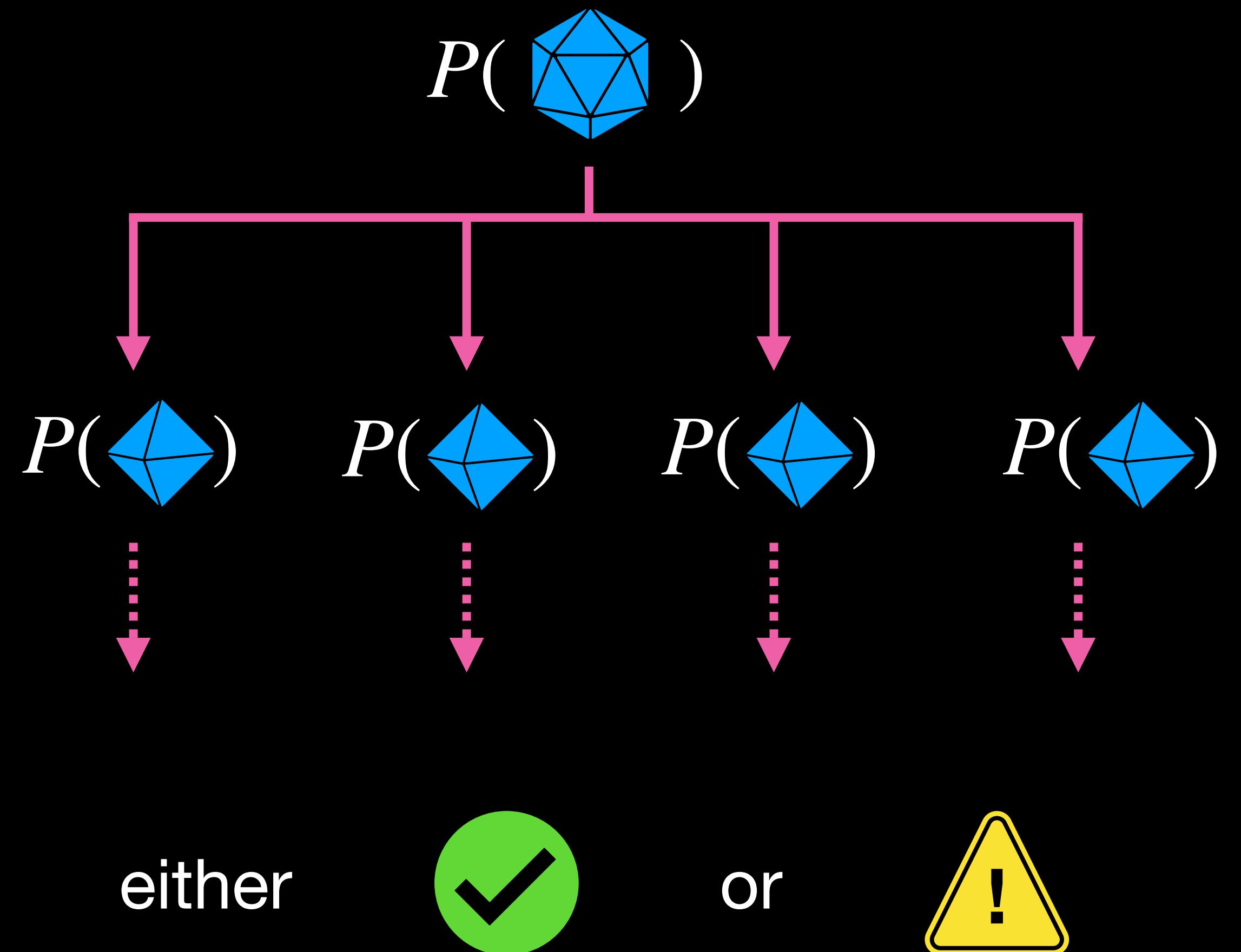
**Dichotomy [Dalvi & Suciu 2012]:**

then  $P(\text{large query expression})$  is in PTIME

then  $P(\text{large query expression})$  is #P-hard

# Boolean Query Evaluation

## Dalvi & Suciu Algorithm

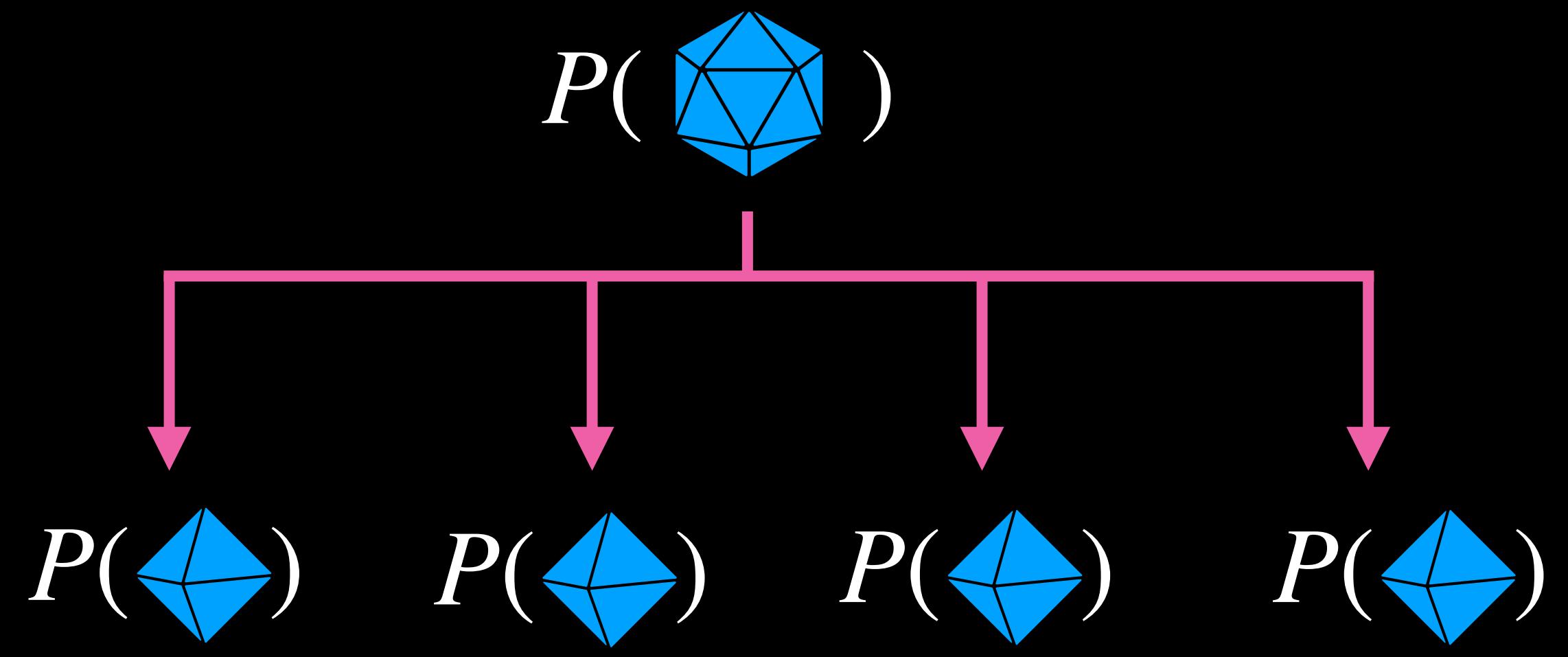


**Dichotomy [Dalvi & Suciu 2012]:**

- then       $P(\text{icosahedron})$  is in PTIME
- then       $P(\text{icosahedron})$  is #P-hard

# Boolean Query Evaluation

# Dalvi & Suciu Algorithm



either



or



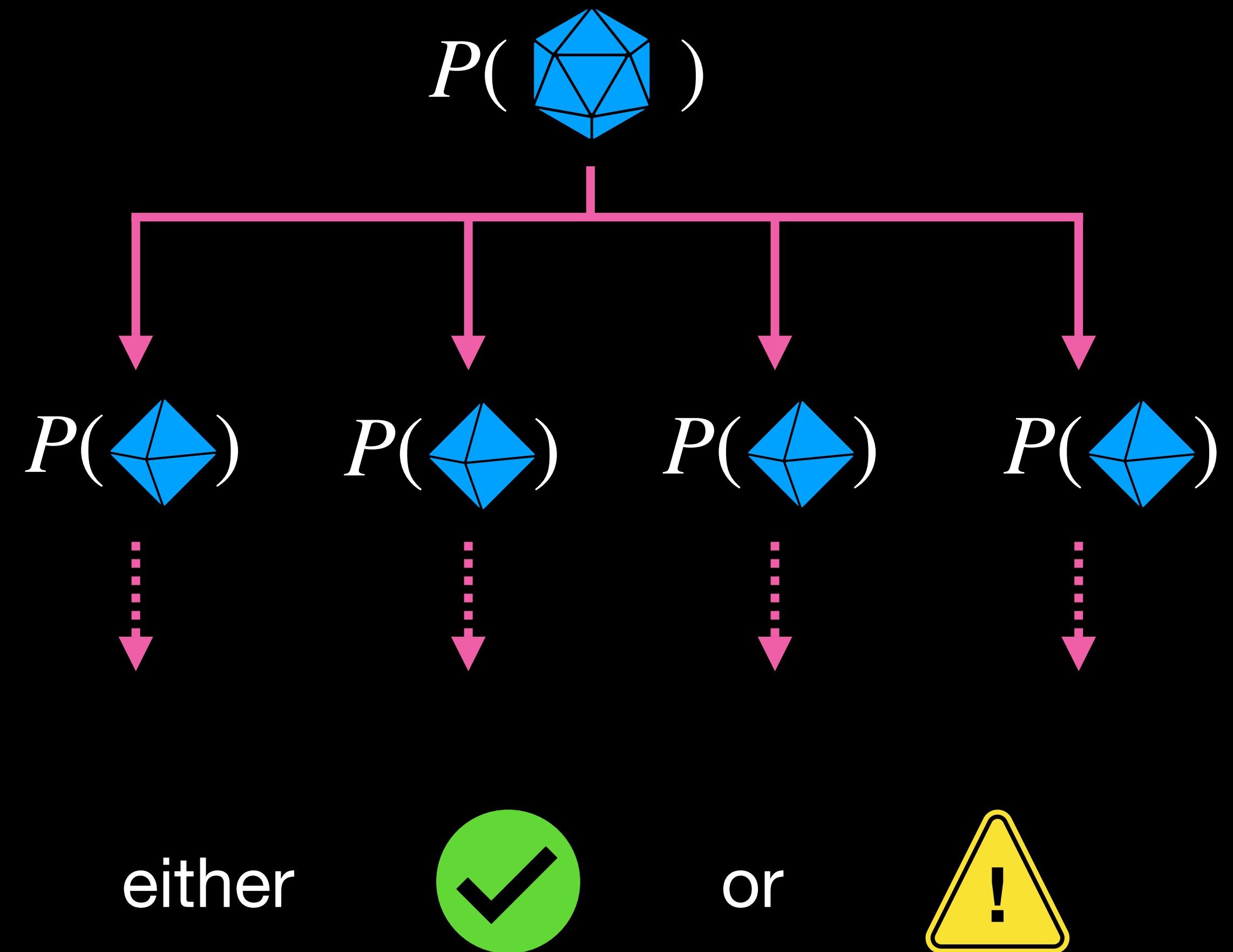
**Dichotomy [Dalvi & Suciu 2012]:**

✓ then  $P(\text{icosahedron})$  is in PTIME

! then  $P(\text{icosahedron})$  is #P-hard

# Boolean Query Evaluation

# Dalvi & Suciu Algorithm



**Dichotomy [Dalvi & Suciu 2012]:**

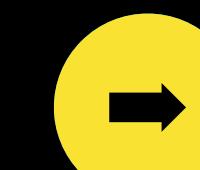
- then  $P(\text{Polyhedron})$  is in PTIME
- then  $P(\text{Polyhedron})$  is #P-hard

**Our Result:**

we can maintain  
the result of this computation  
under single-tuple updates  
with constant update time



# Probabilistic Databases under Updates



Boolean Query Evaluation  
and Ranked Enumeration

4

Christoph Berkholz, Maximilian Merz

Talk at AIMoTh 2022, based on the PODS 2021 paper



# Probabilistic Databases under Updates



-  Boolean Query Evaluation
-  4 and Ranked Enumeration

Christoph Berkholz, Maximilian Merz

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# Probabilistic Databases under Updates



Boolean Query Evaluation  
and Ranked Enumeration

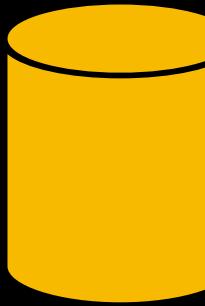
Christoph Berkholz, Maximilian Merz

Talk at AIMoTh 2022, based on the PODS 2021 paper

# Updates & Enumeration

init

- fix db schema
- fix finite domain
- fix FO query



insert

insert

insert

answer

insert

delete

answer

...

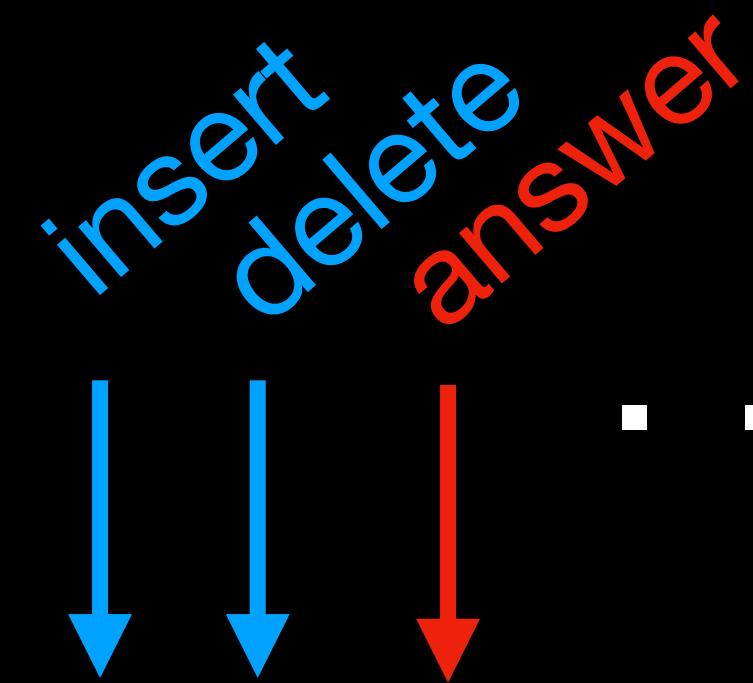
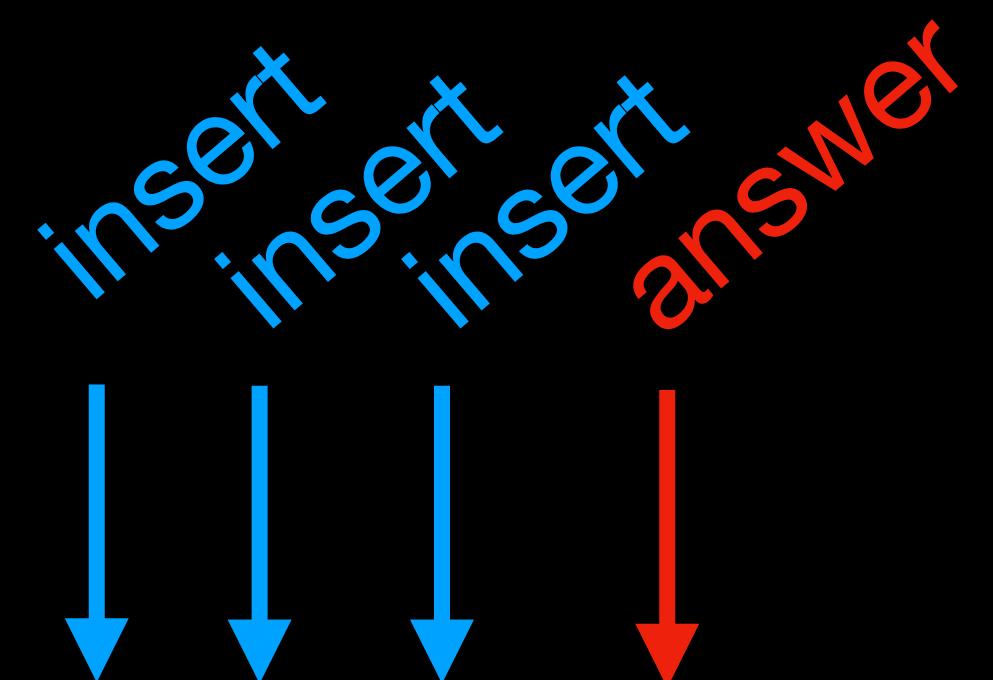
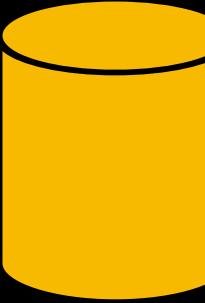
time



# Updates & Enumeration

init

- fix db schema
- fix finite domain
- fix FO query

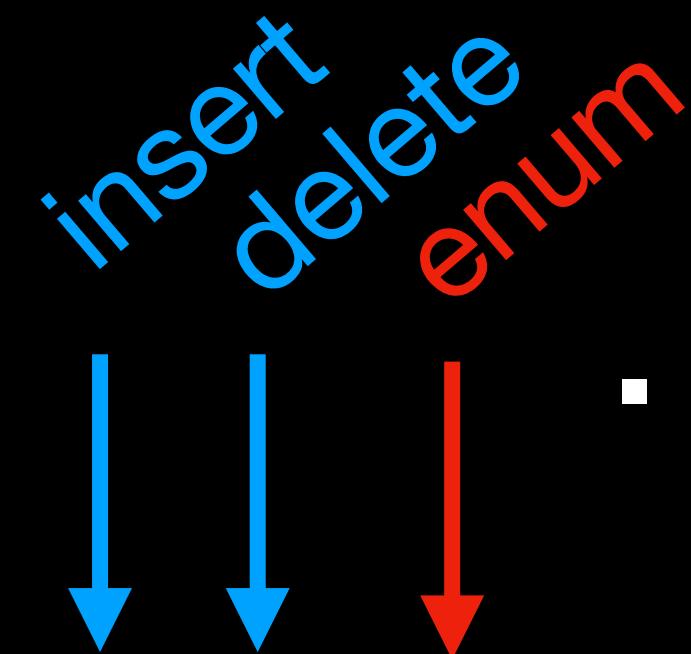
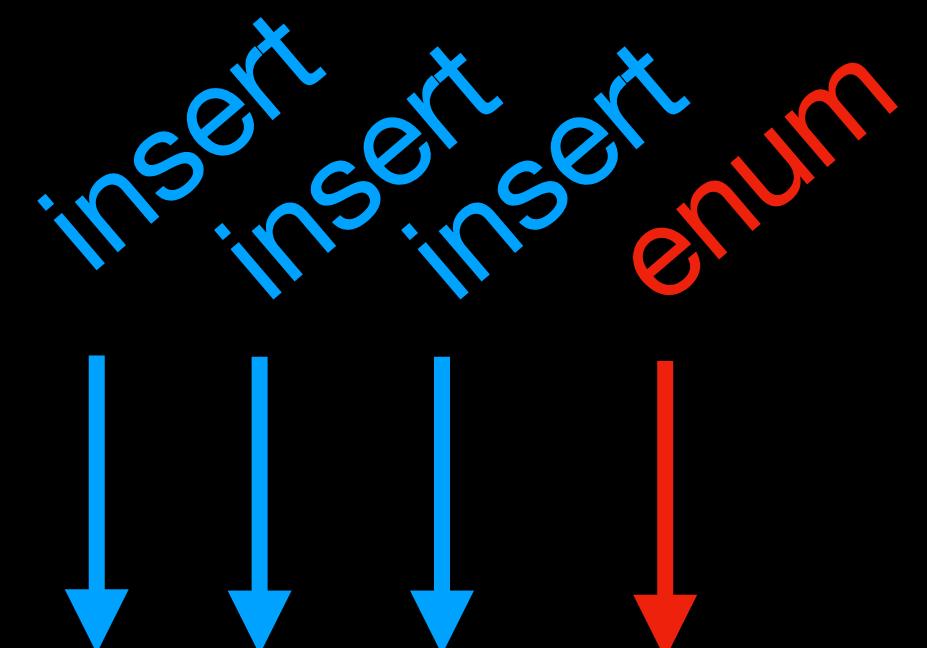
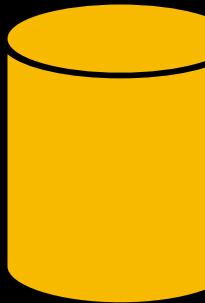


time

# Updates & Enumeration

init

- fix db schema
- fix finite domain
- fix FO query

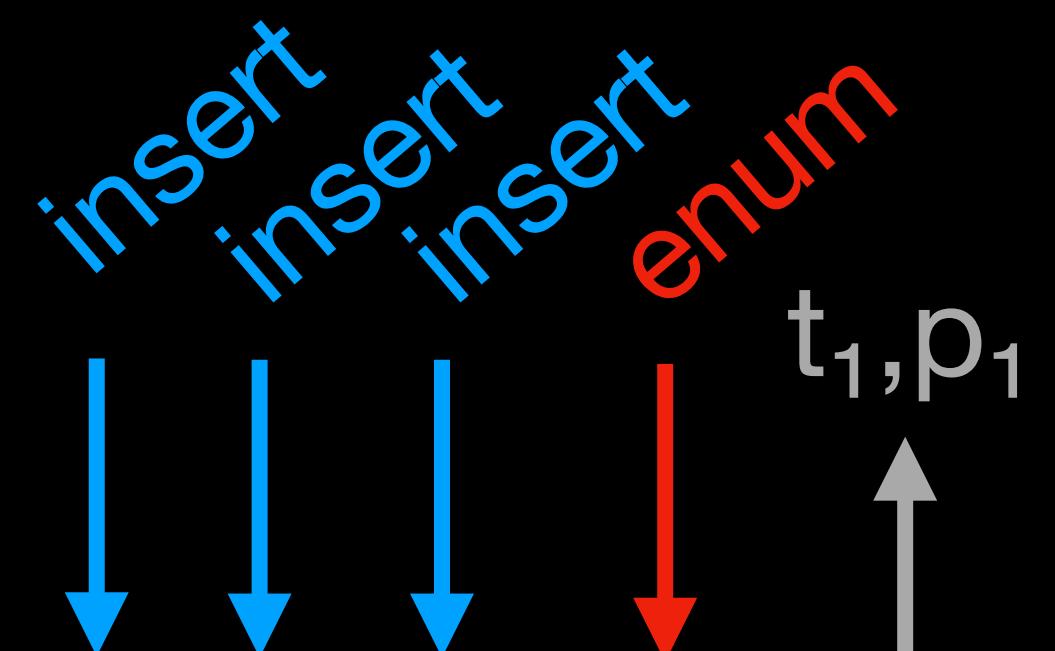
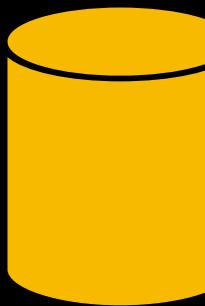


time

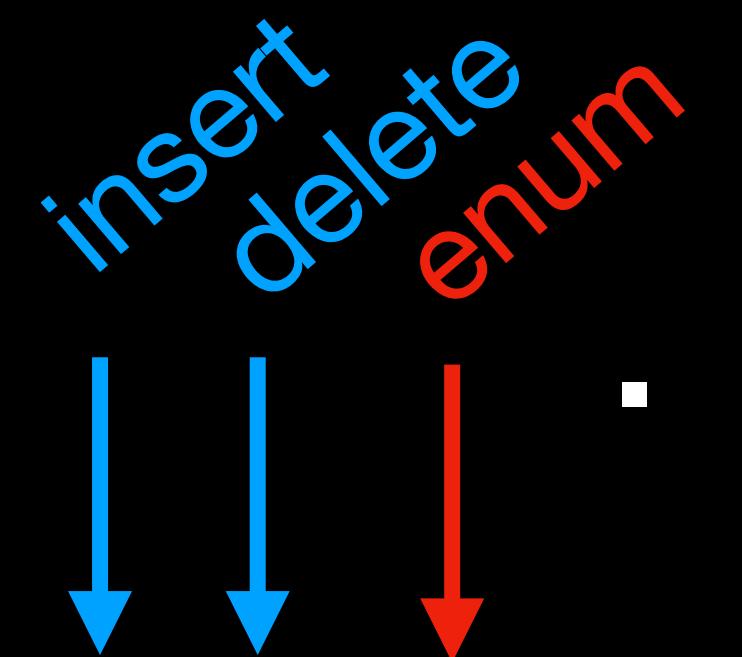
# Updates & Enumeration

init

- fix db schema
- fix finite domain
- fix FO query



$t_1, p_1$

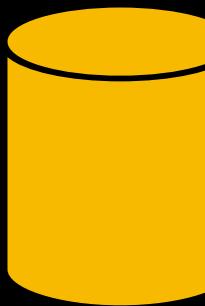


time

# Updates & Enumeration

init

- fix db schema
- fix finite domain
- fix FO query



insert  
insert  
insert

enum

$t_1, p_1$

next

$t_2, p_2$

insert  
delete  
enum

...

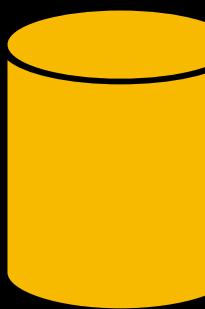
time



# Updates & Enumeration

init

- fix db schema
- fix finite domain
- fix FO query



insert  
insert  
insert

enum

t<sub>1</sub>,p<sub>1</sub>

next

t<sub>2</sub>,p<sub>2</sub>

next

t<sub>3</sub>,p<sub>3</sub>

...

insert  
delete  
enum

...

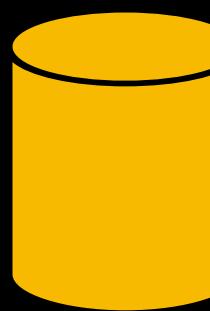
time



# Updates & Enumeration

init

- fix db schema
- fix finite domain
- fix FO query



insert  
insert  
insert  
enum

$t_1, p_1$

next

$t_2, p_2$

next

$t_3, p_3$

next

EoE

insert  
delete  
enum

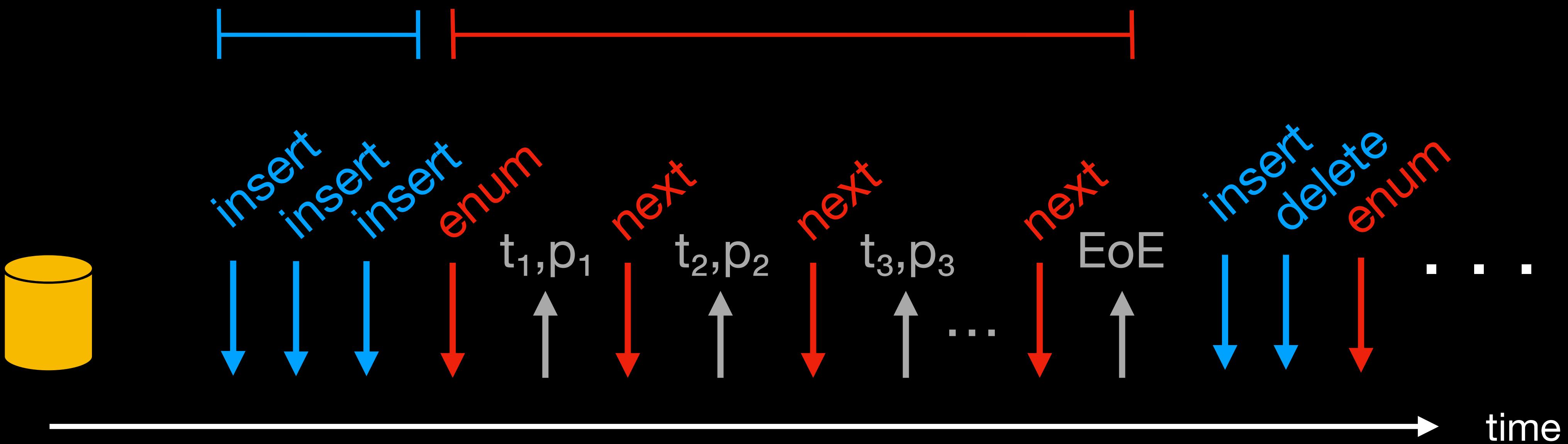
time

# Updates & Enumeration

init

- fix db schema
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- fix FO query

- update phase: insert / delete
- enumeration phase

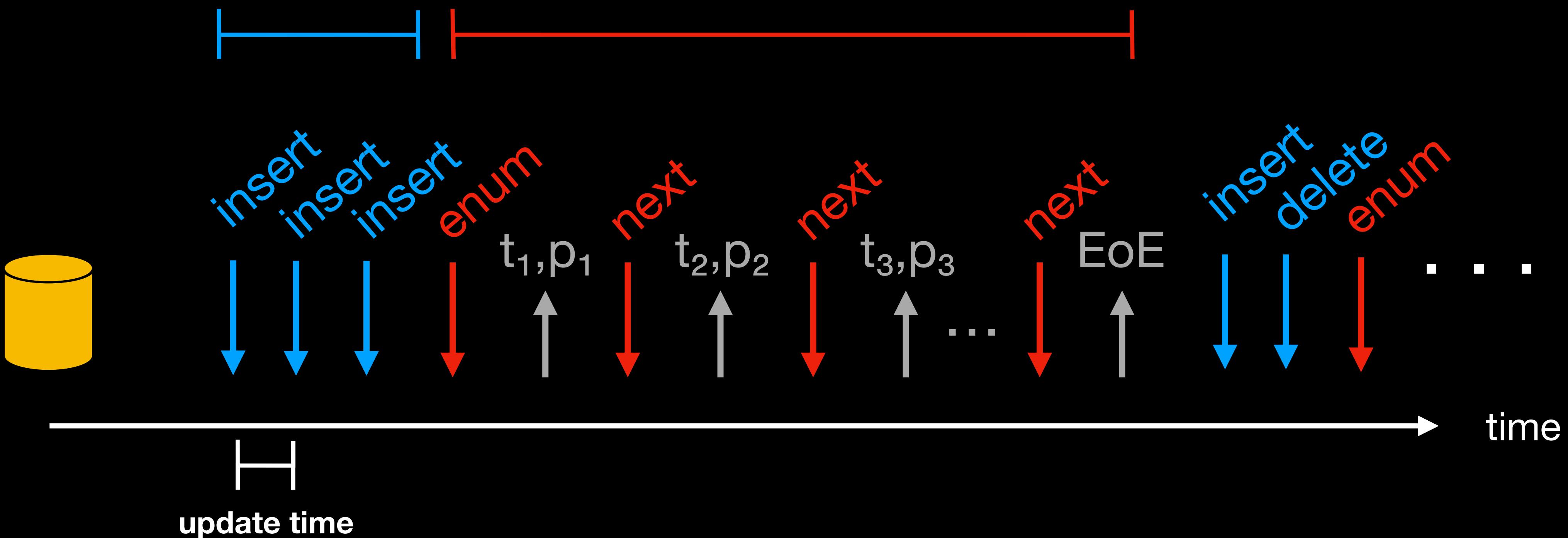


# Updates & Enumeration

init

- fix db schema
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- fix FO query

- update phase: insert / delete
- enumeration phase

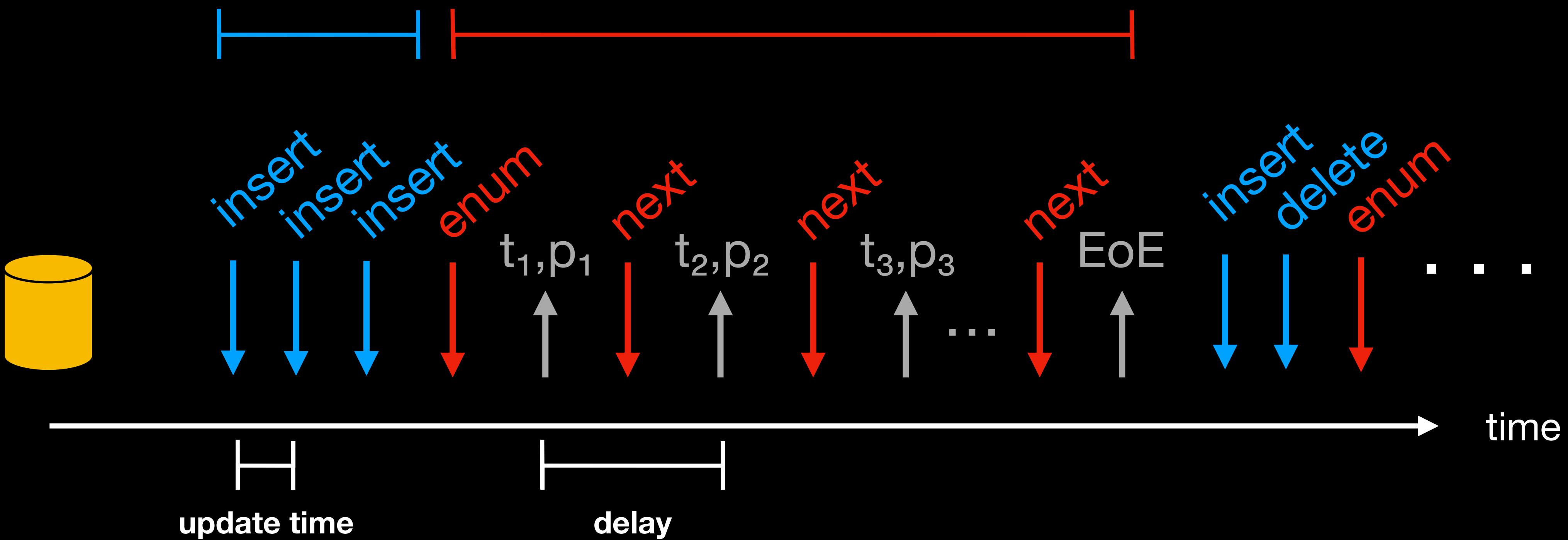


# Updates & Enumeration

init

- fix db schema
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- fix FO query

- update phase: insert / delete
- enumeration phase

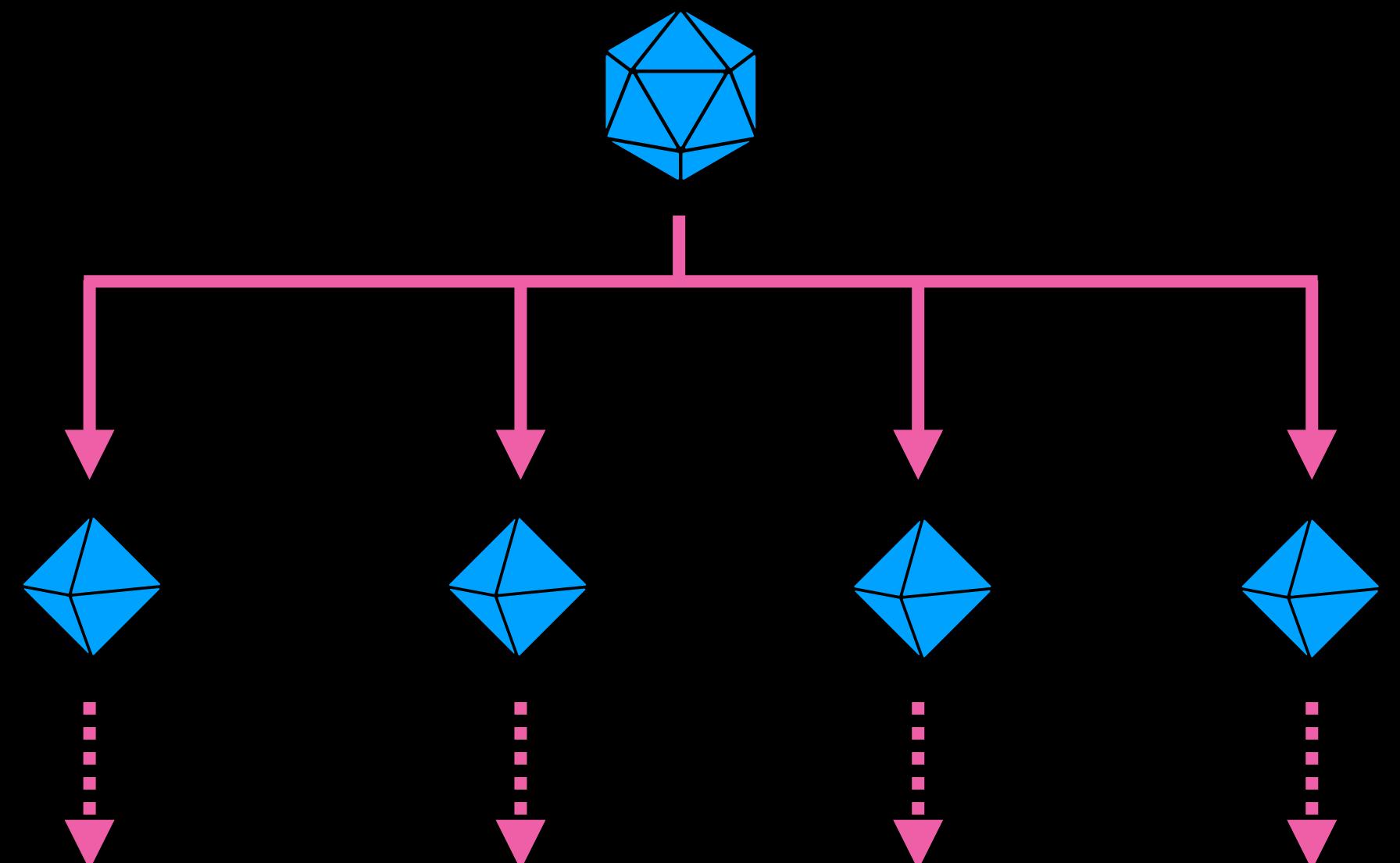


# Ranked Enumeration: Our Algorithm

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**Berkholz, Keppeler, Schweikardt:**  
**(2017)**

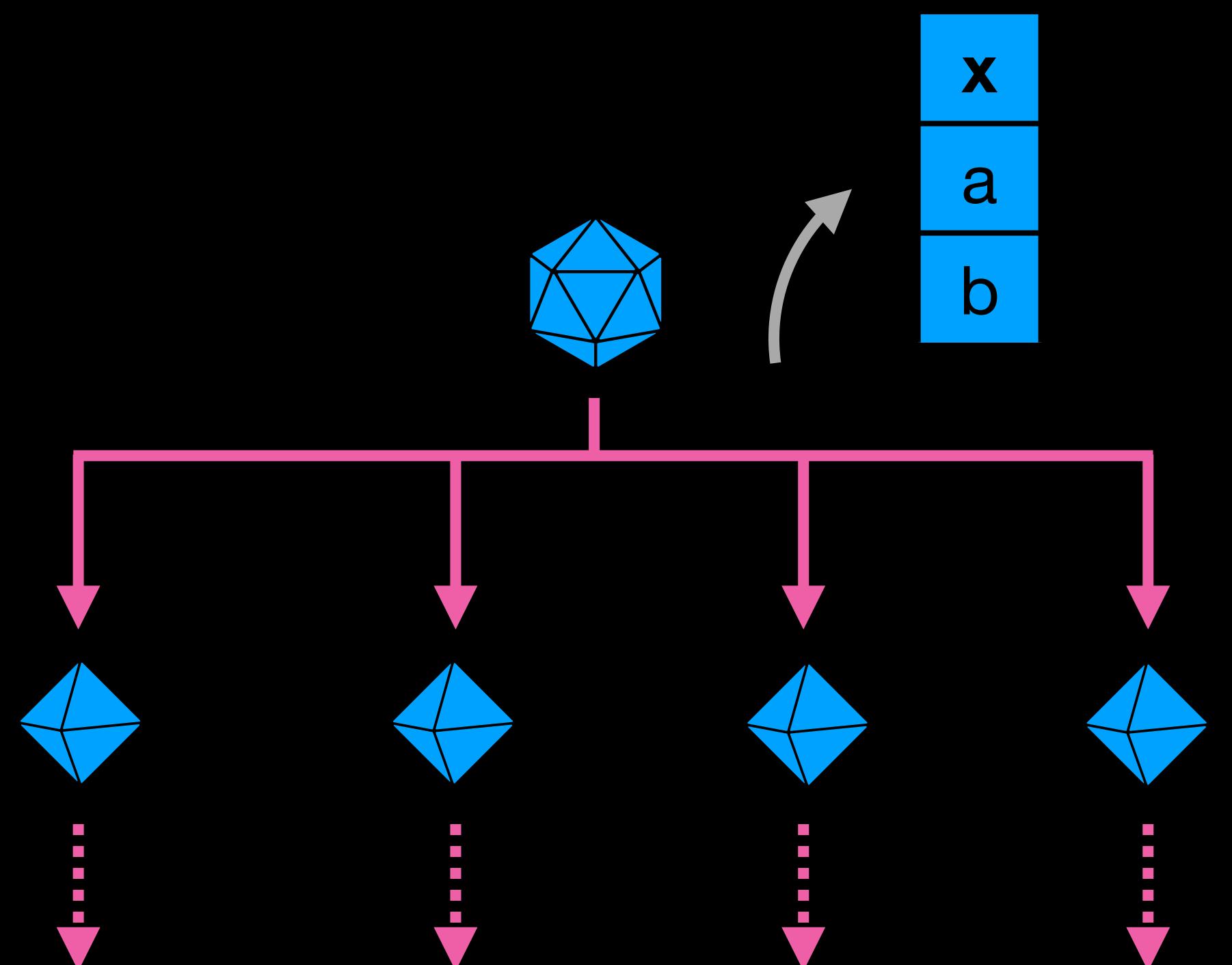
...on deterministic DB's



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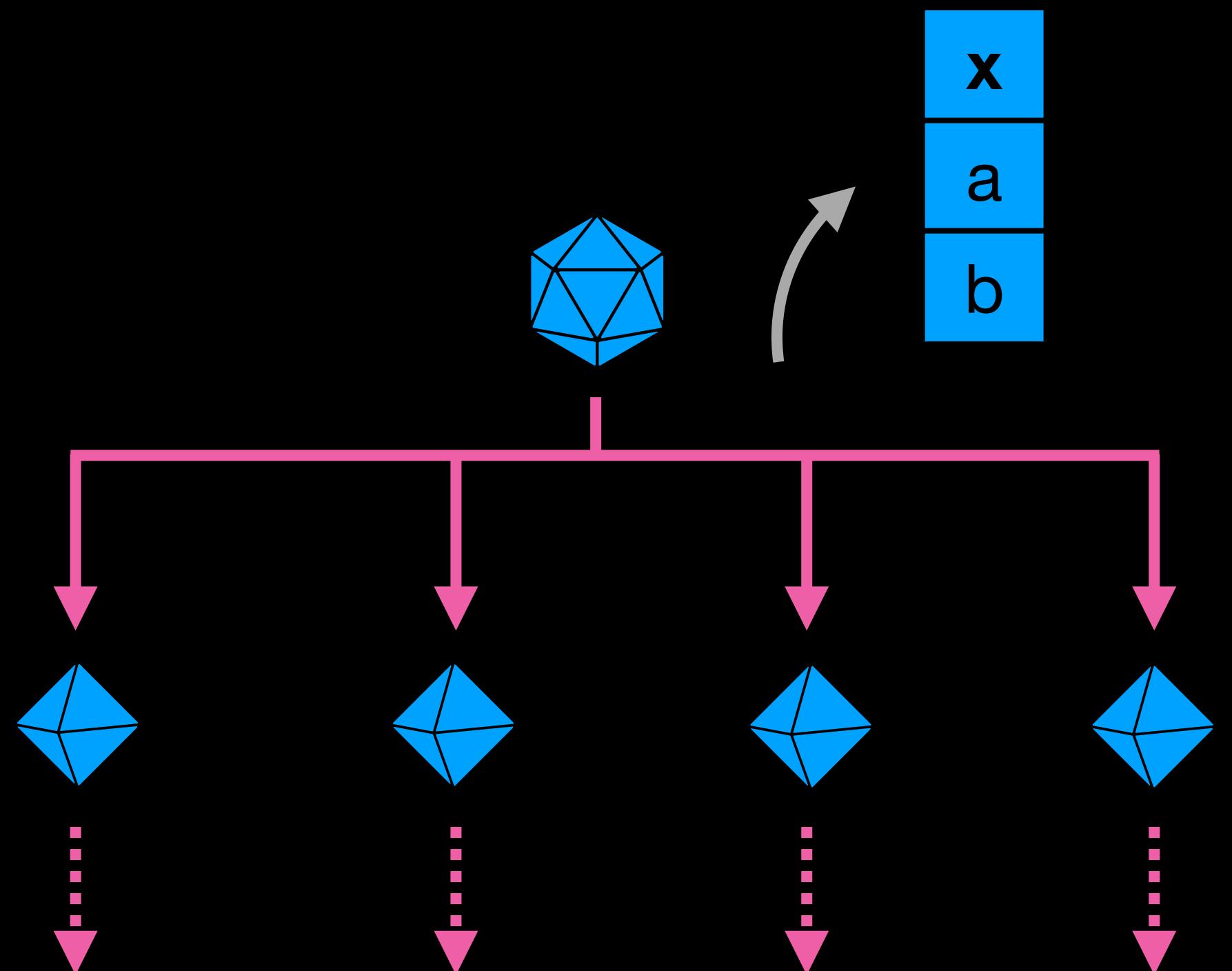
# Ranked Enumeration: Our Algorithm

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**Our Result:**

...on tuple-independent  
probabilistic DB's



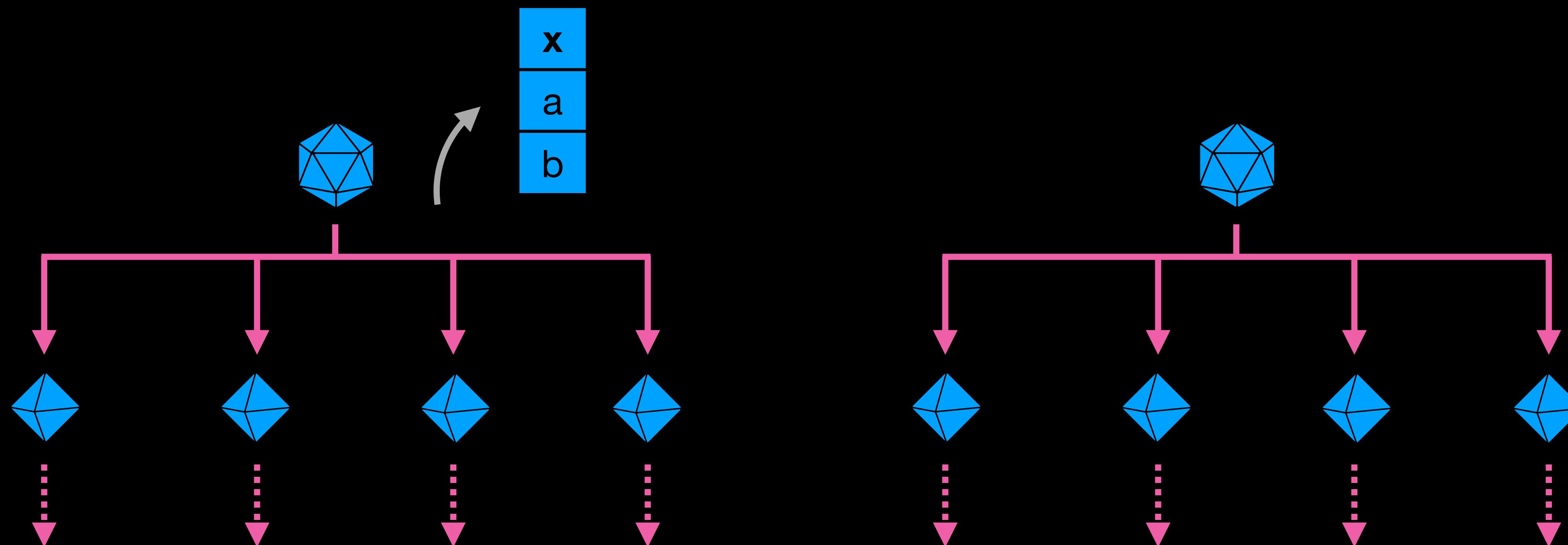
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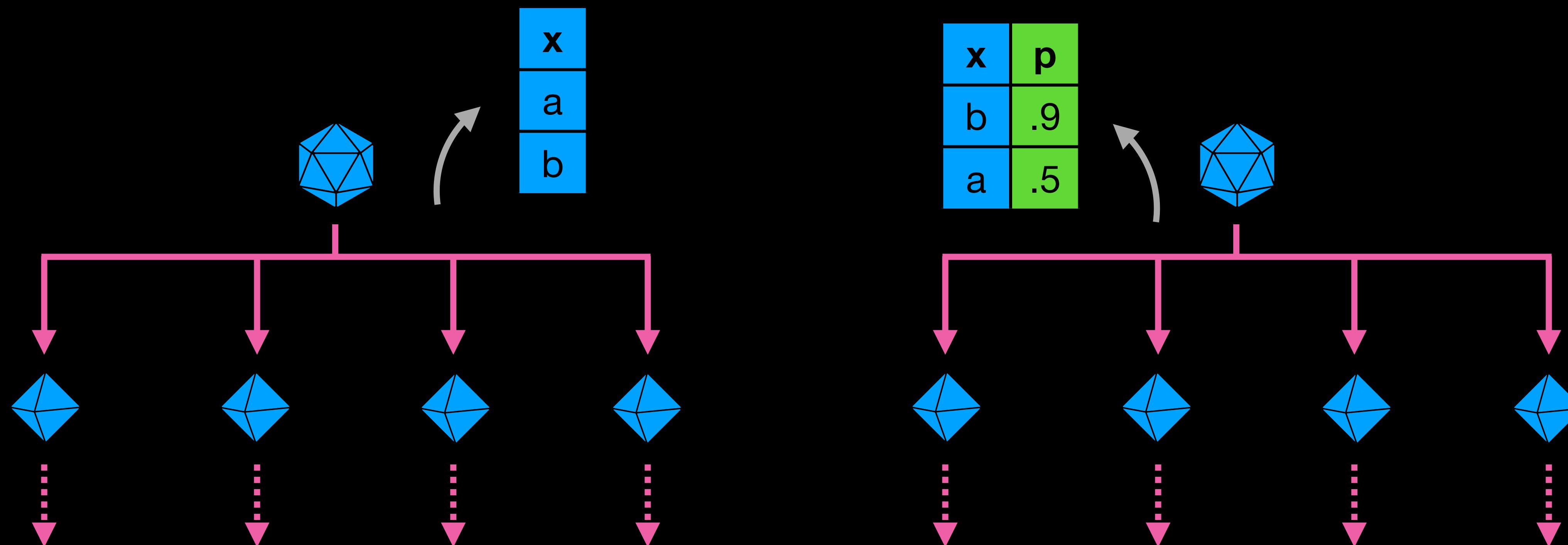
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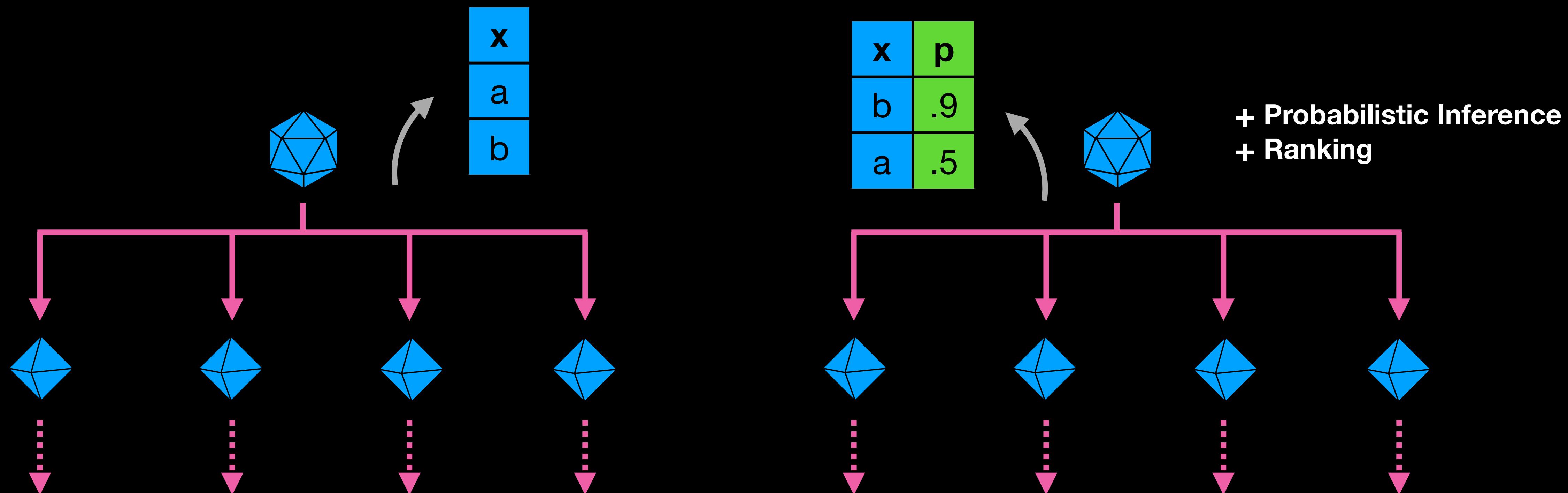
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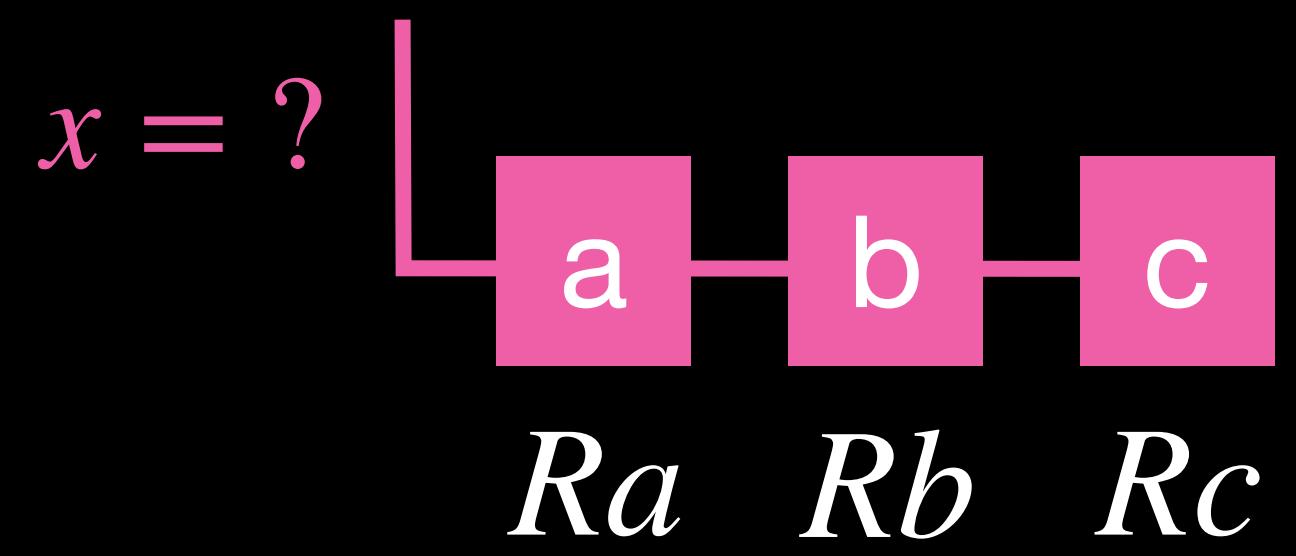
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**Our Result:**

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probabilistic DB's

$$\varphi(x) = Rx$$



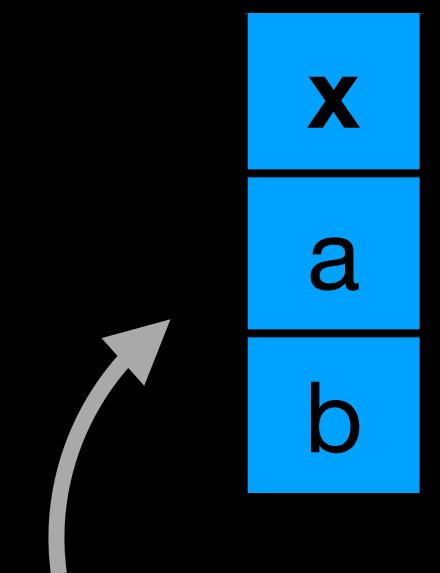
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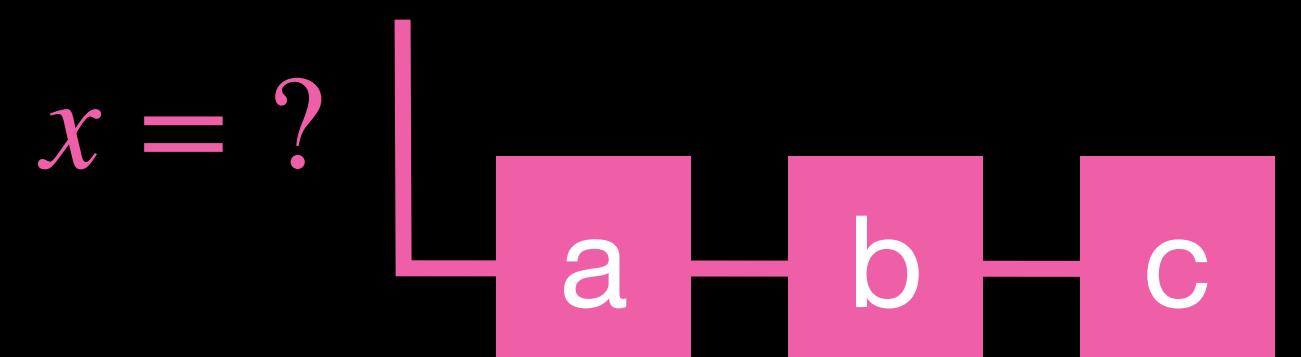
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$$\varphi(x) = Rx$$



$$Ra \quad Rb \quad Rc$$

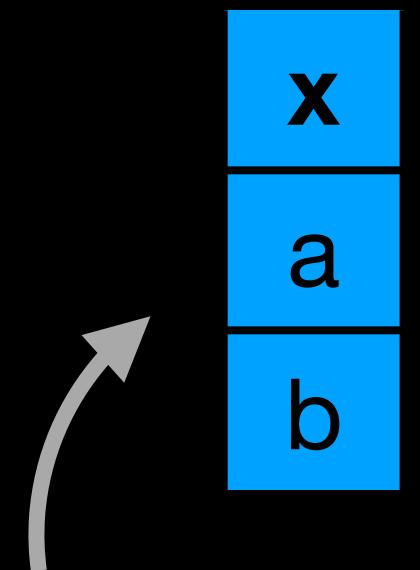
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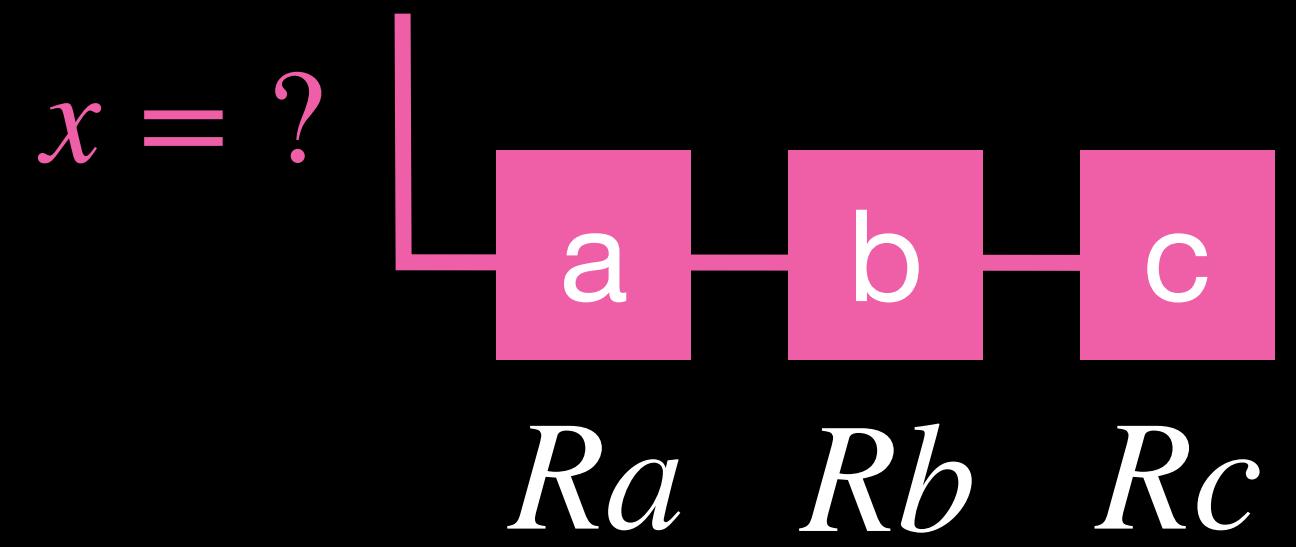
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**Our Result:**

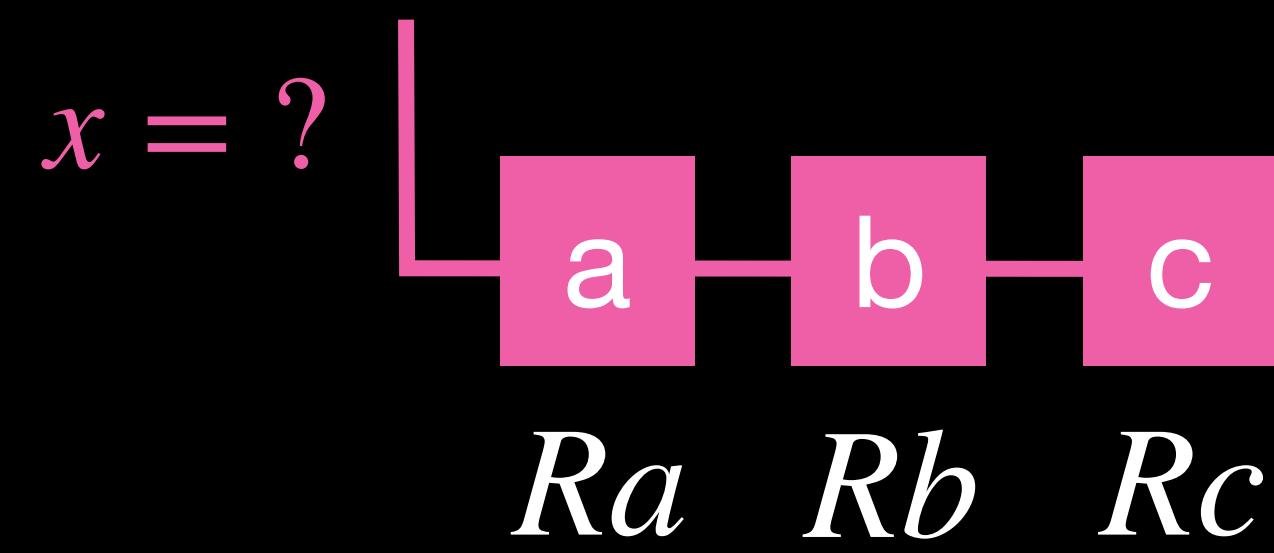
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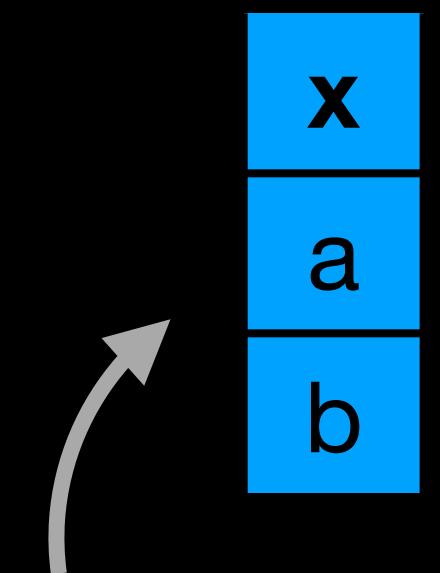
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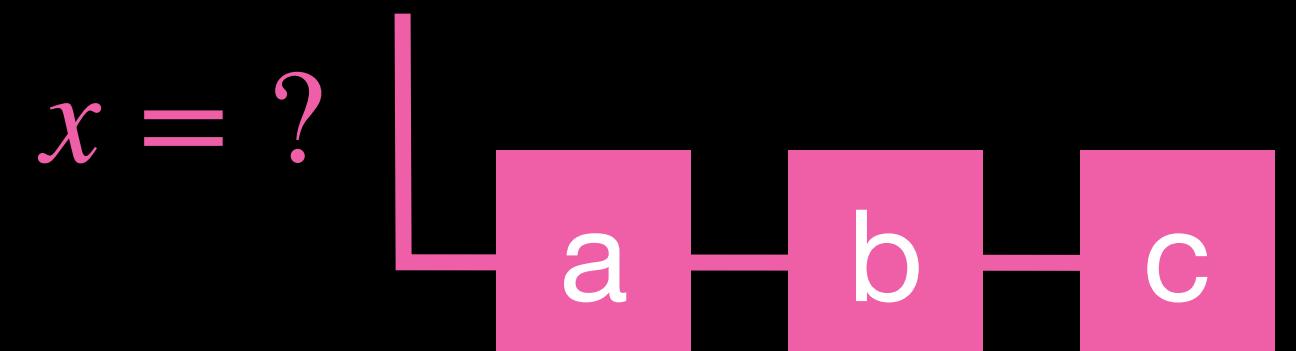
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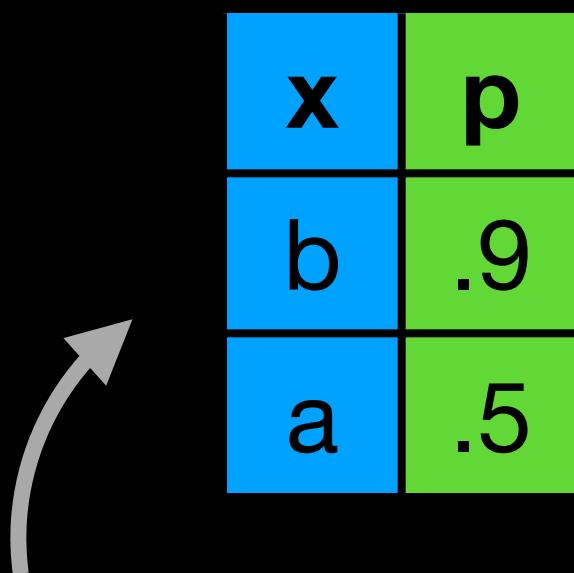
...on tuple-independent  
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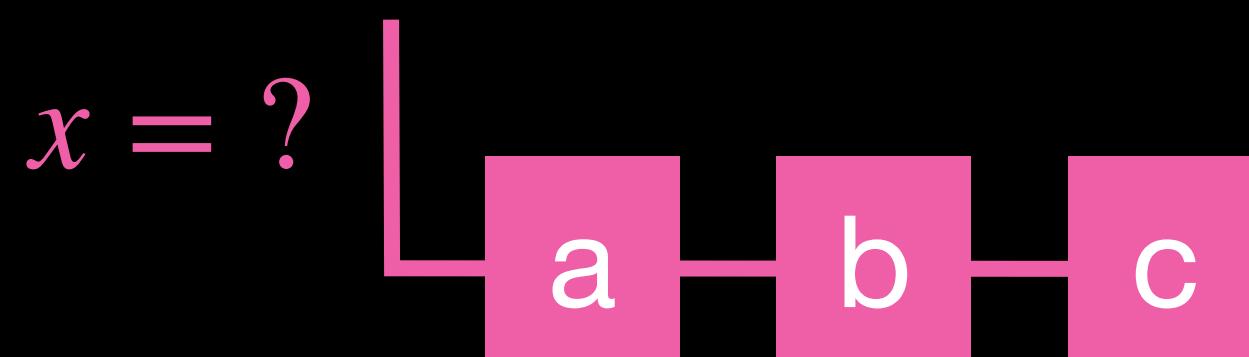
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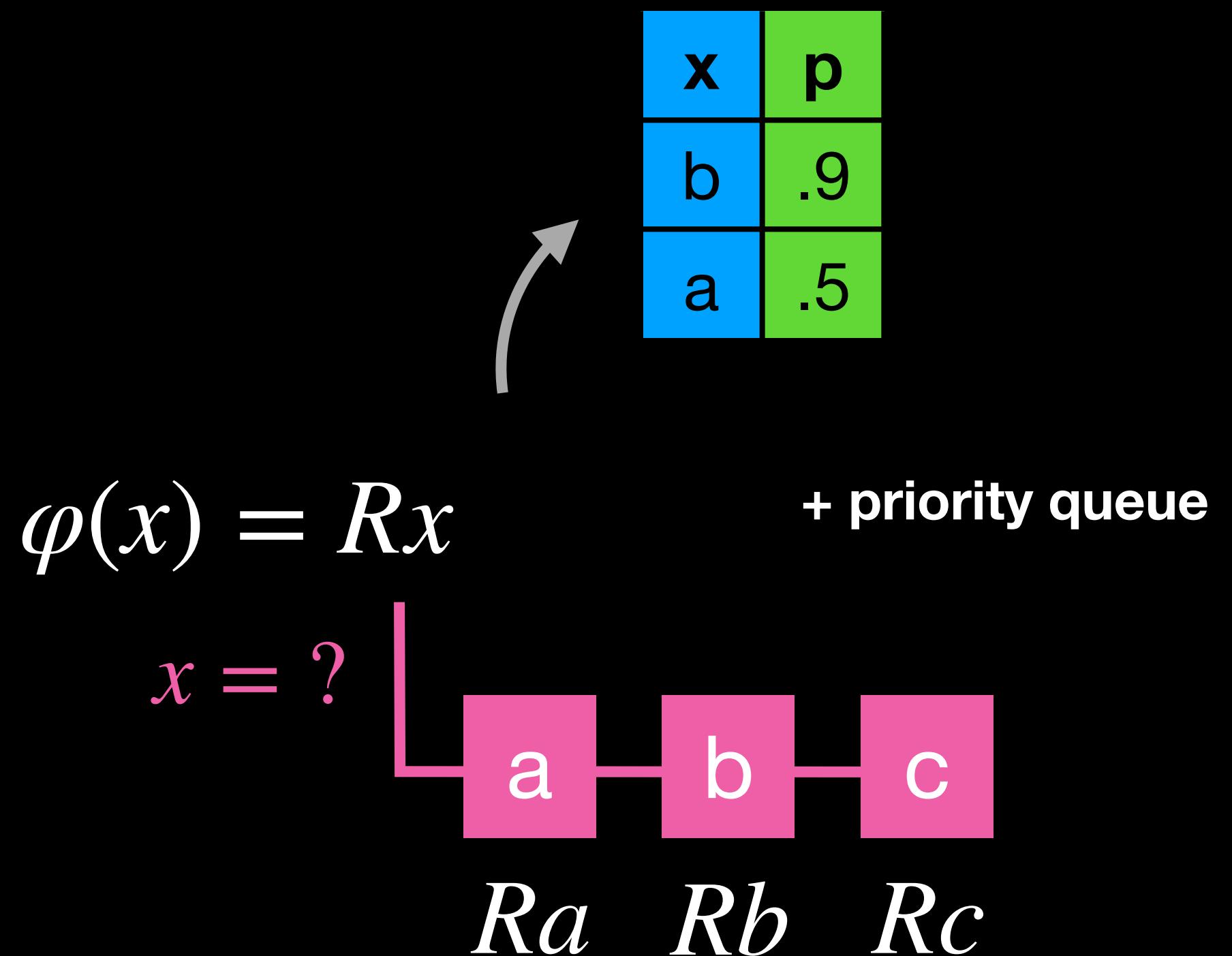
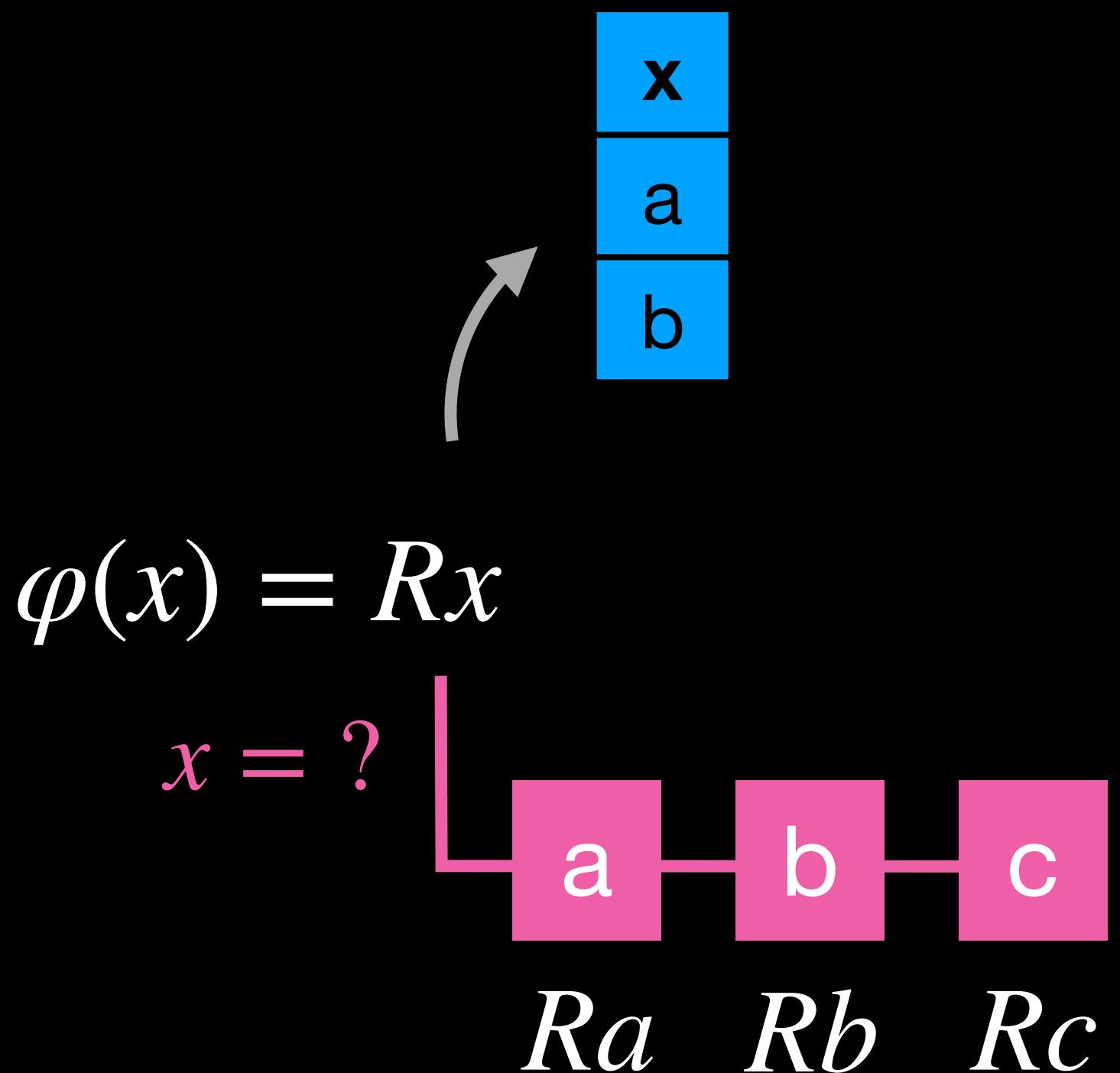
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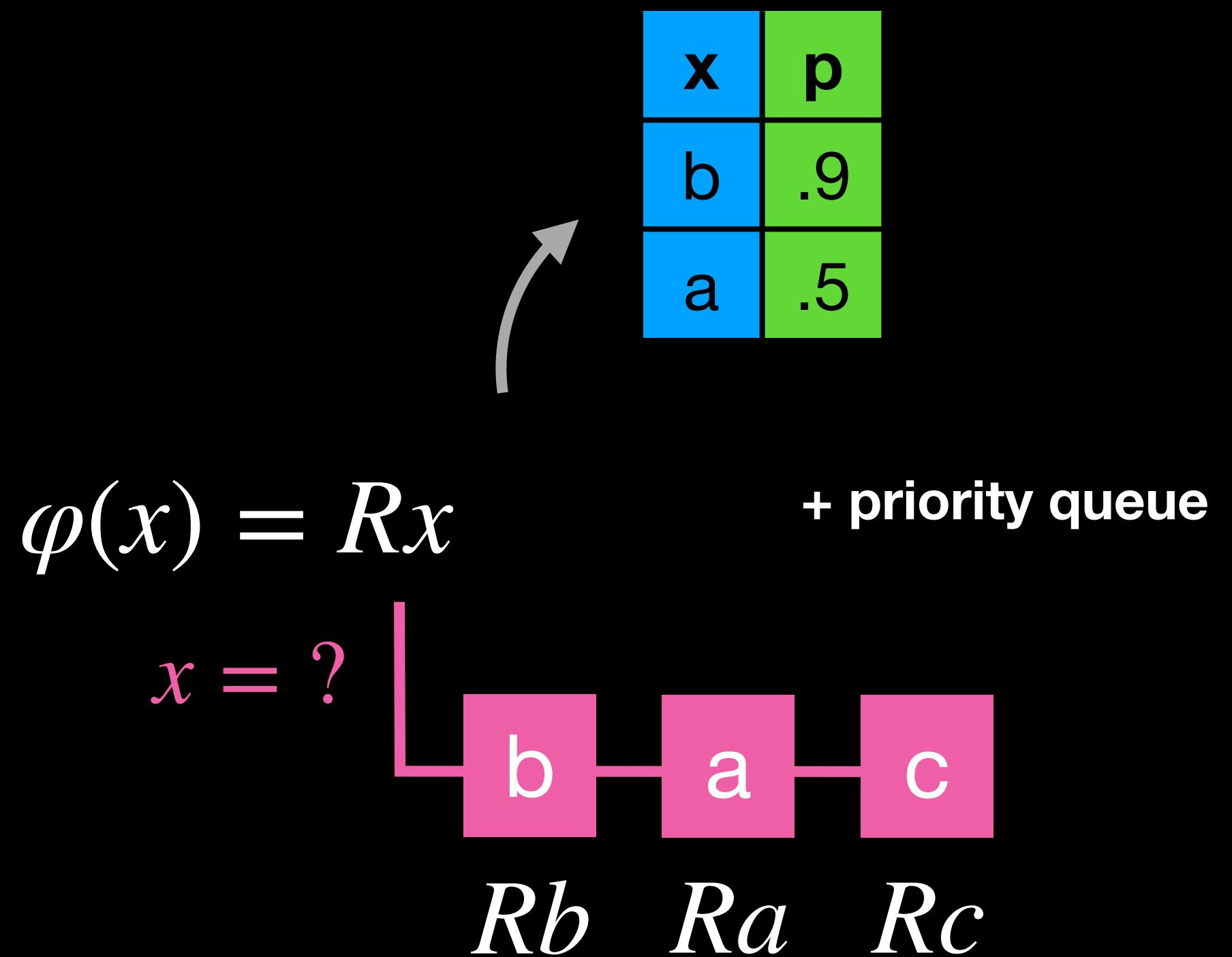
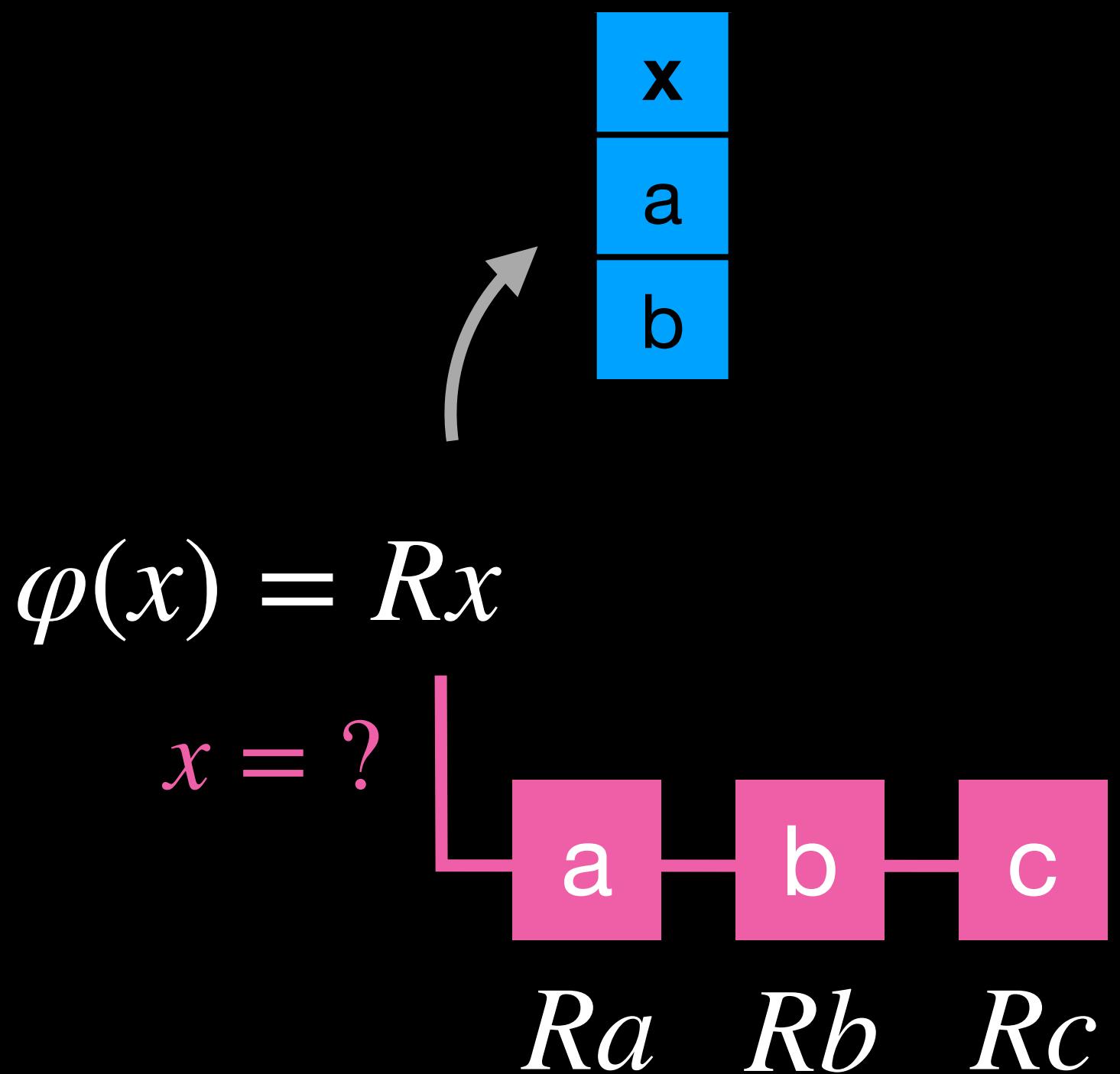
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$$\varphi(x, y) = Rx \wedge Sy$$

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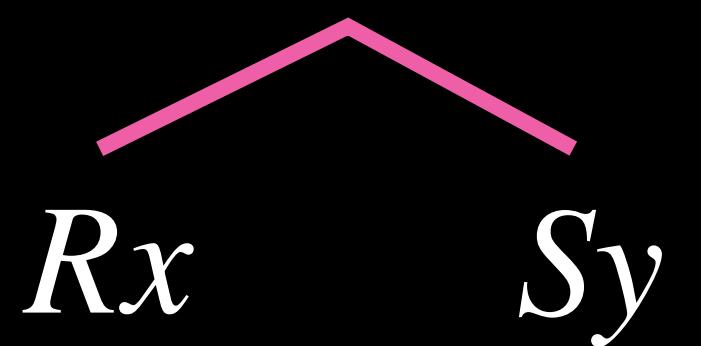
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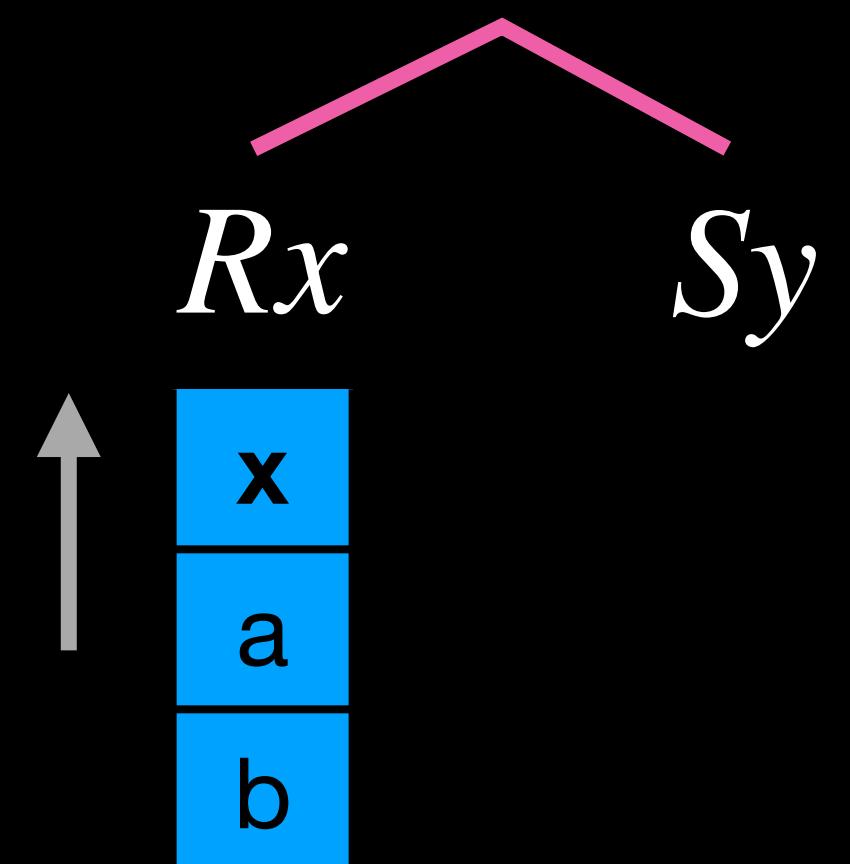
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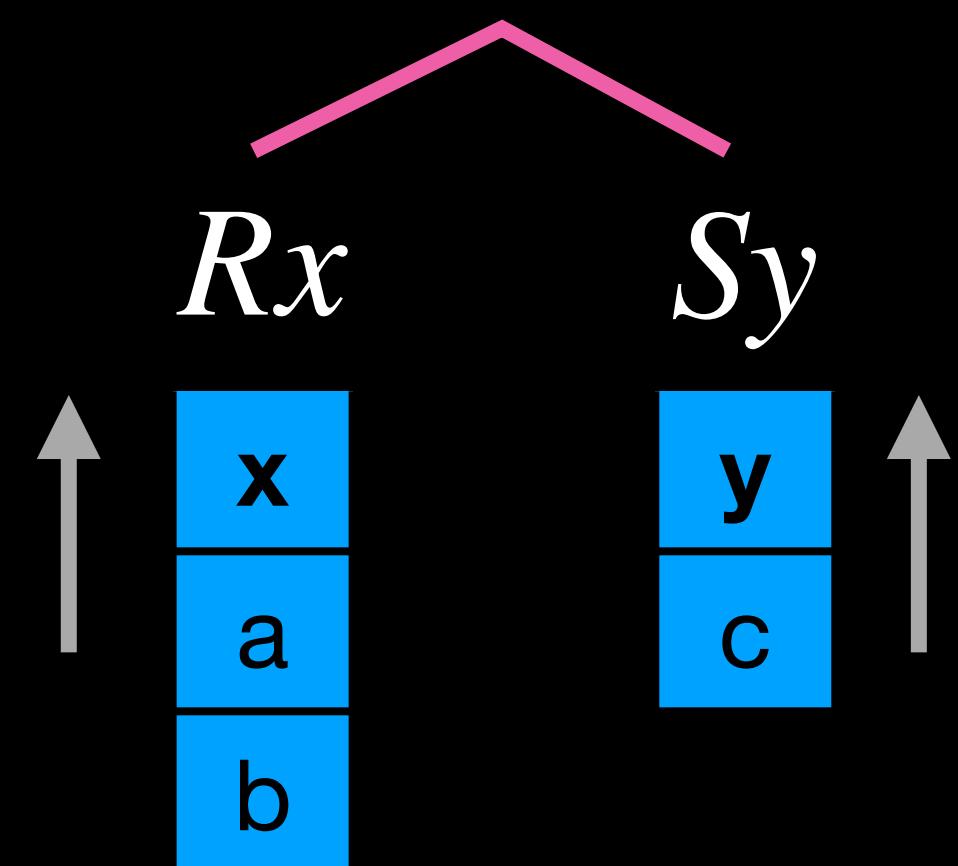
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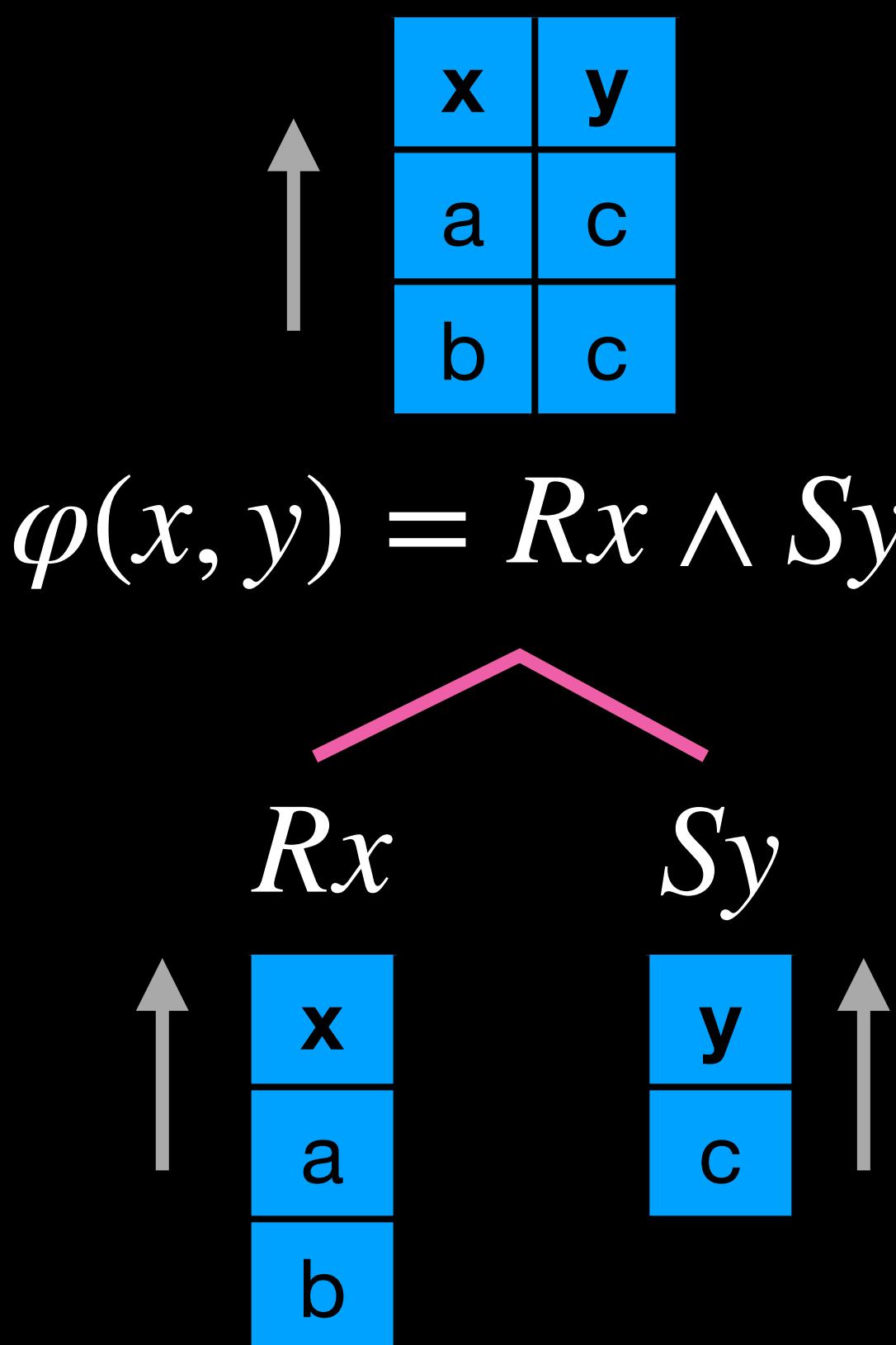
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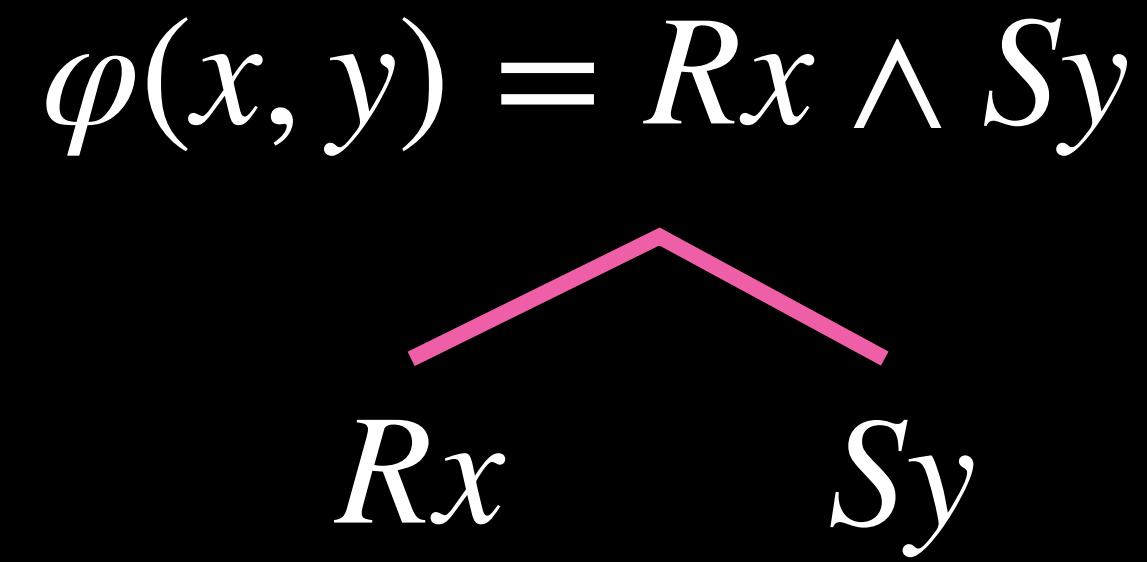
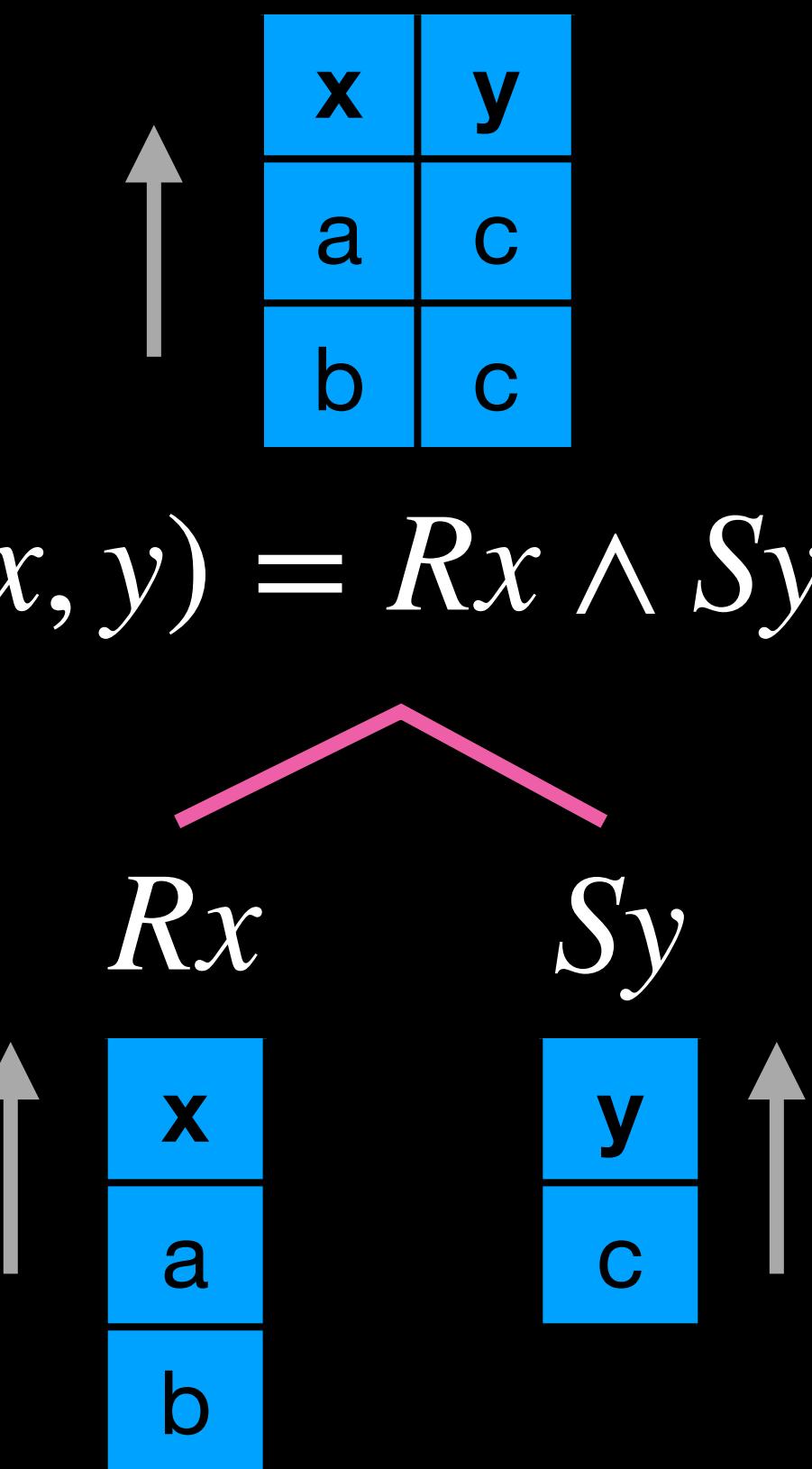
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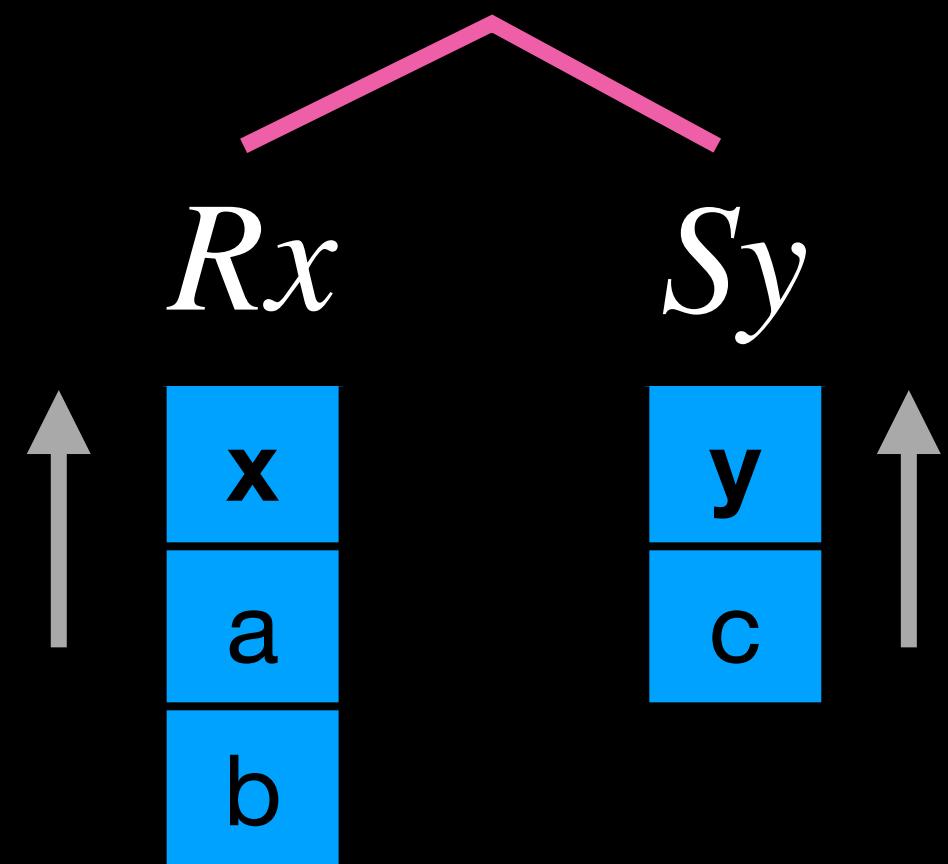
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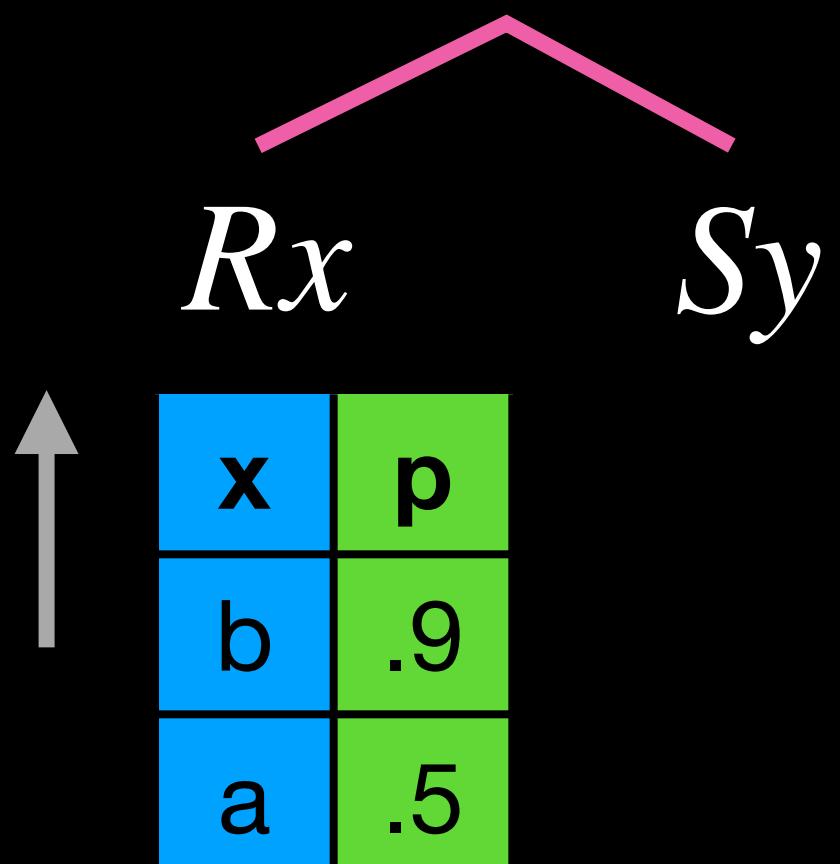
...on tuple-independent  
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x	y
a	c
b	c

$$\varphi(x, y) = Rx \wedge Sy$$



$$\varphi(x, y) = Rx \wedge Sy$$



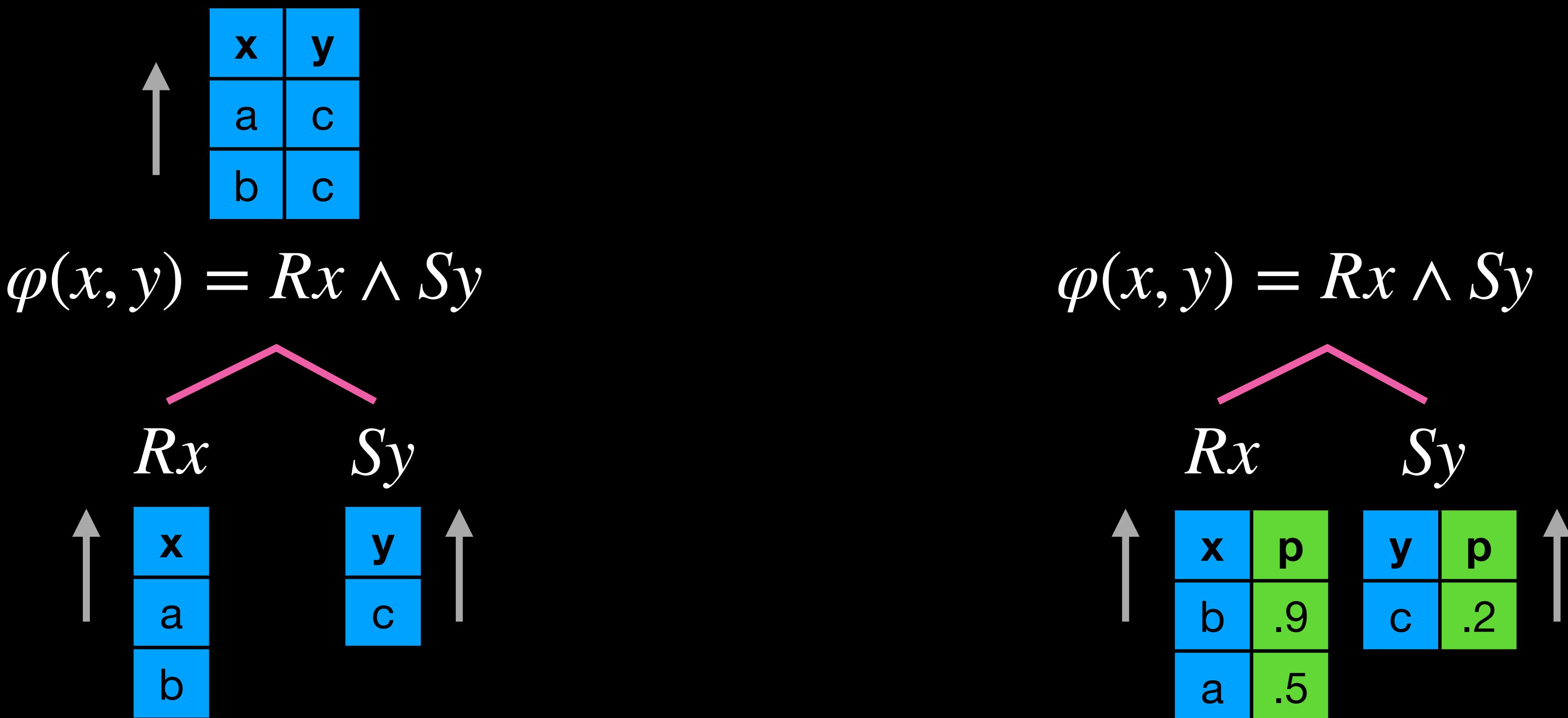
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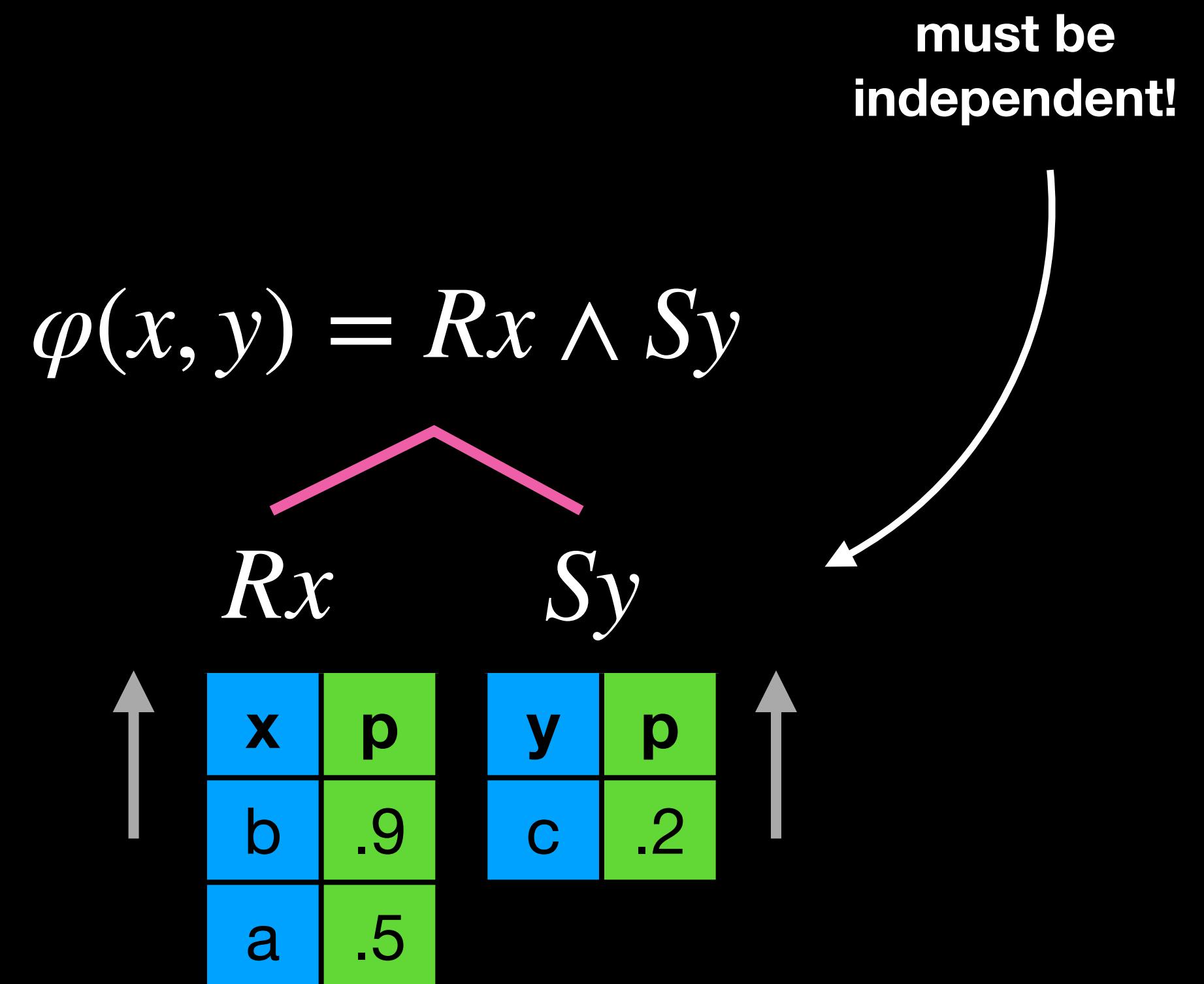
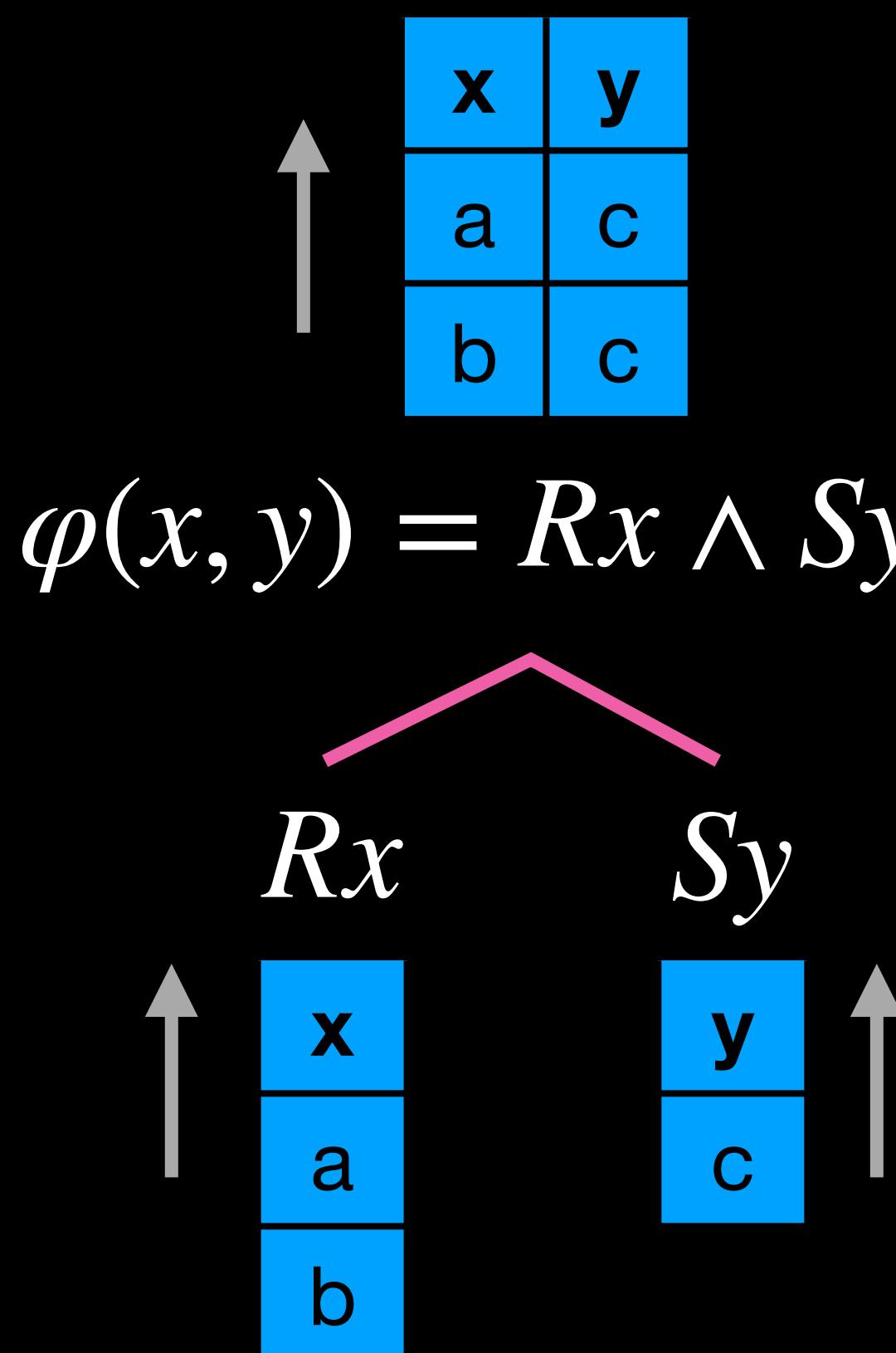
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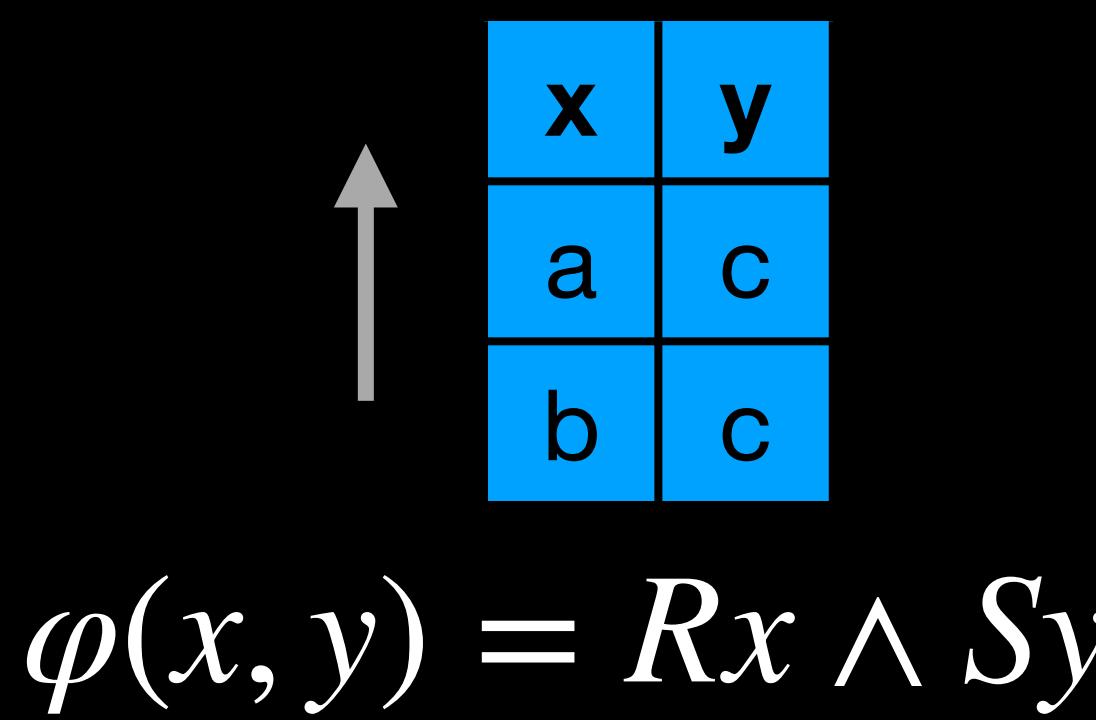
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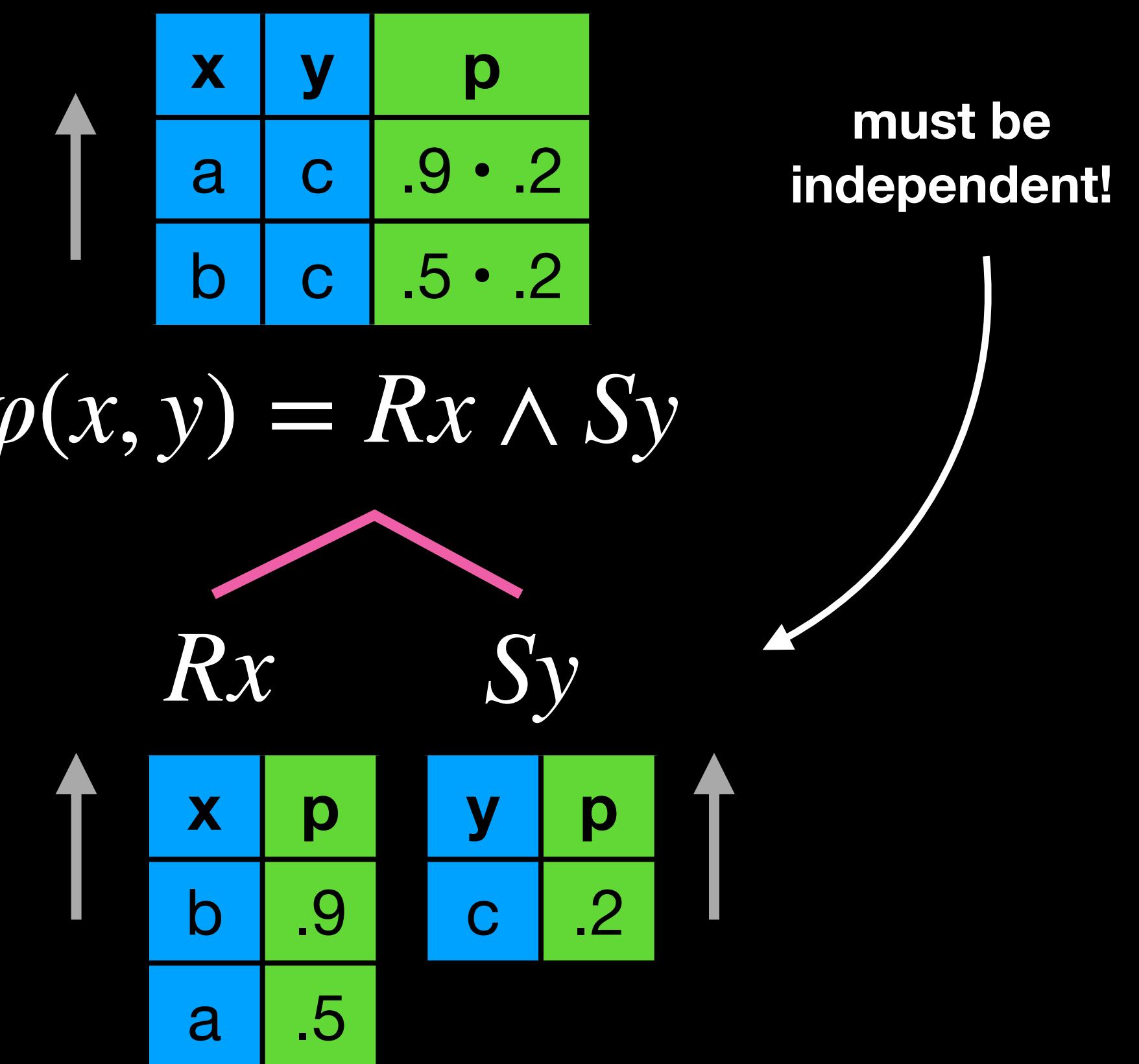
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Our Result:

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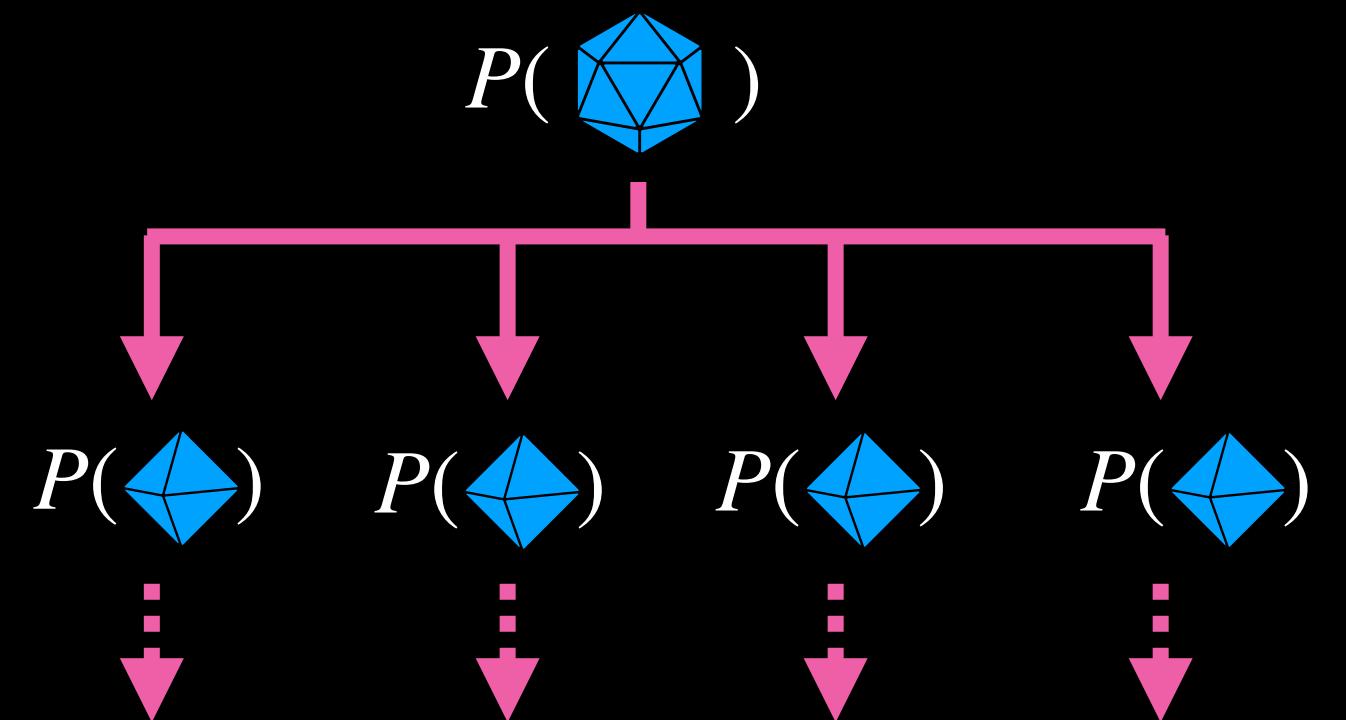
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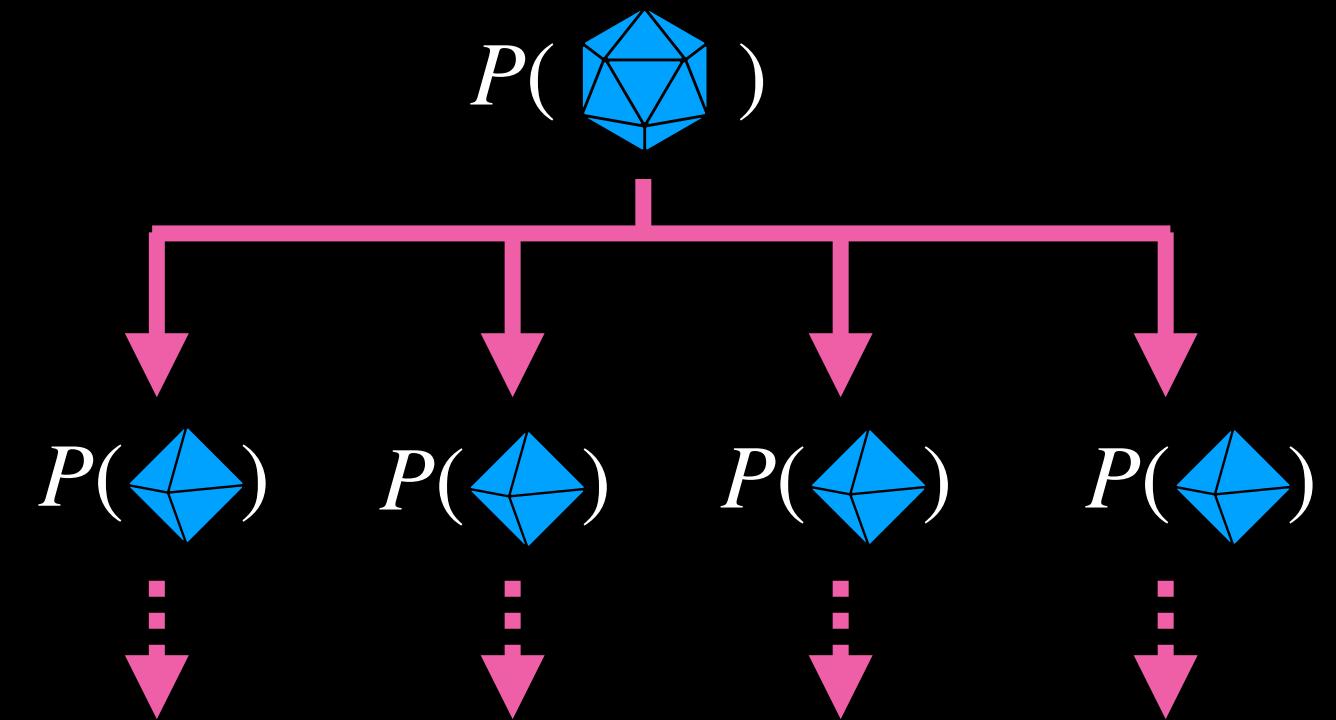
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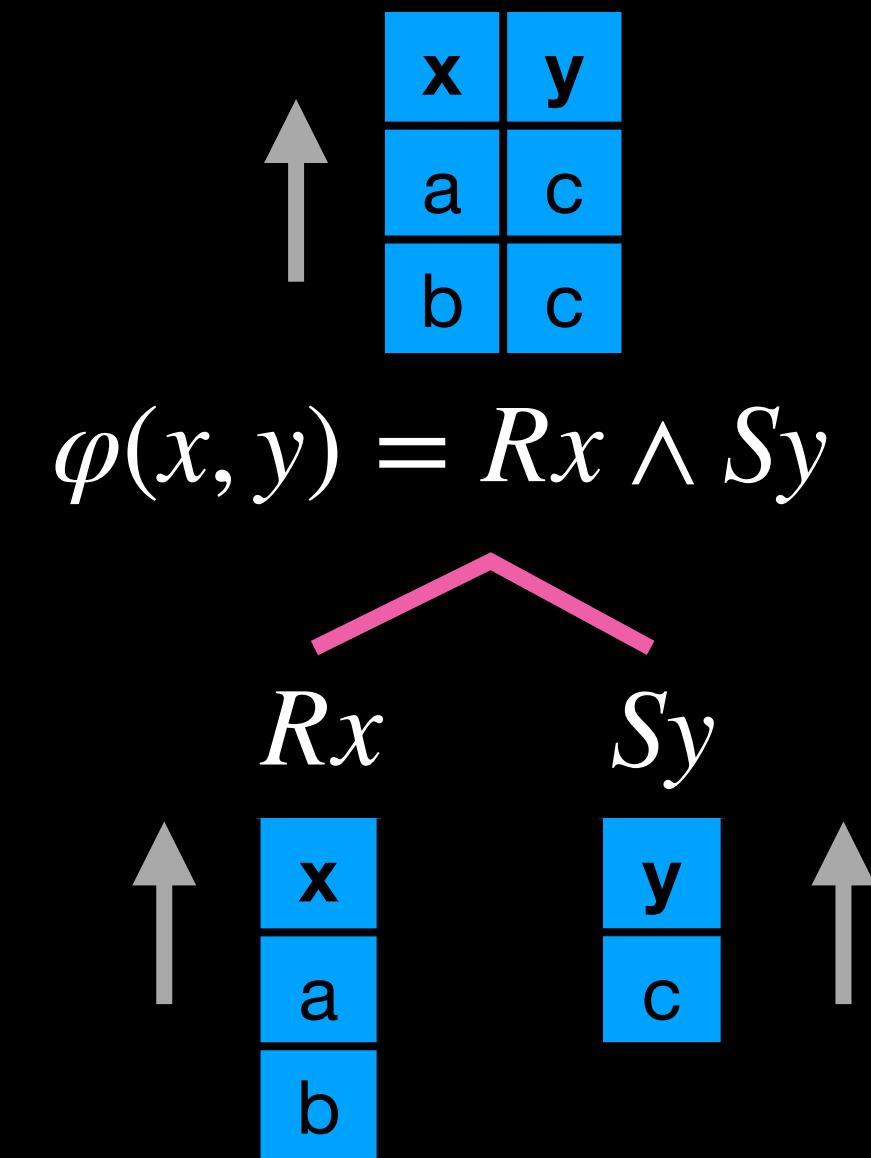
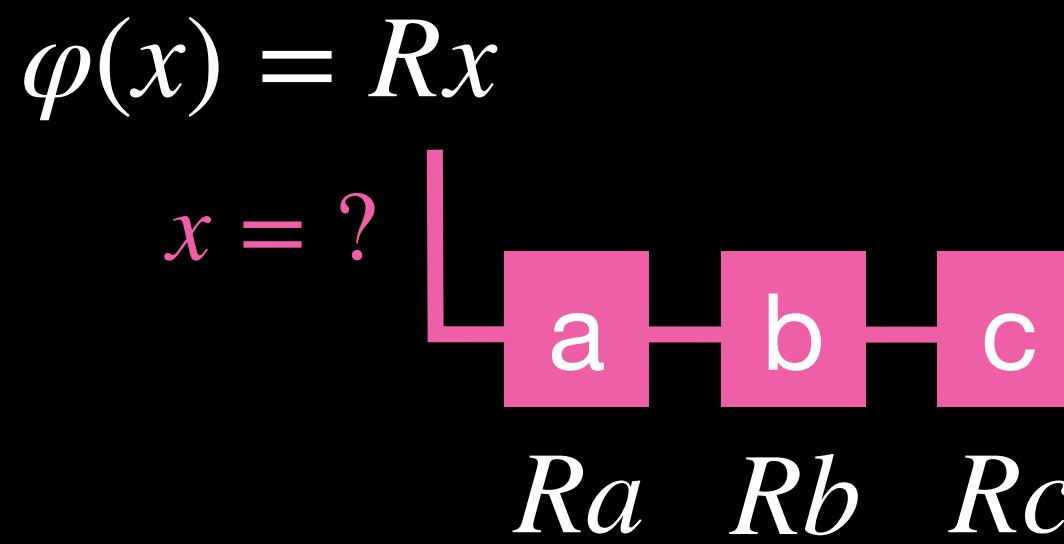
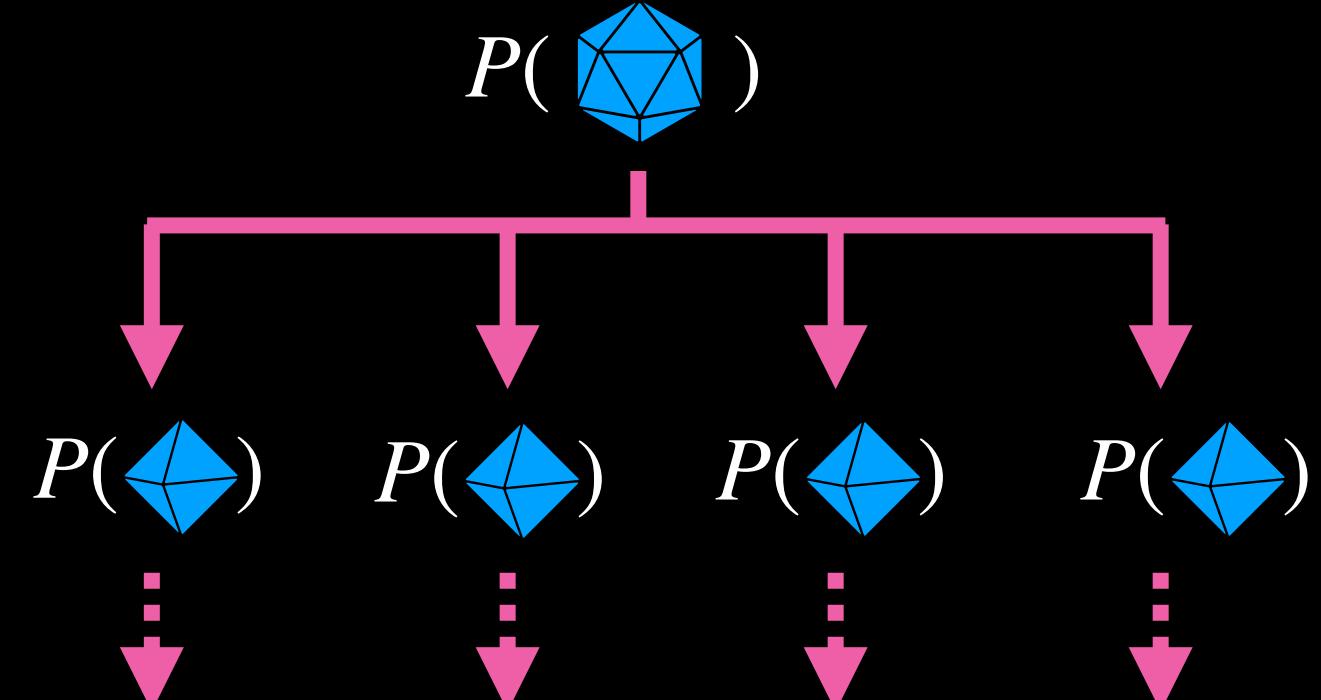
$$\varphi(x) = Rx$$
$$x = ?$$

```
graph LR; a[Ra] --> b[Rb]; b --> c[Rc]
```

**Berkholz, Keppeler, Schweikardt:**  
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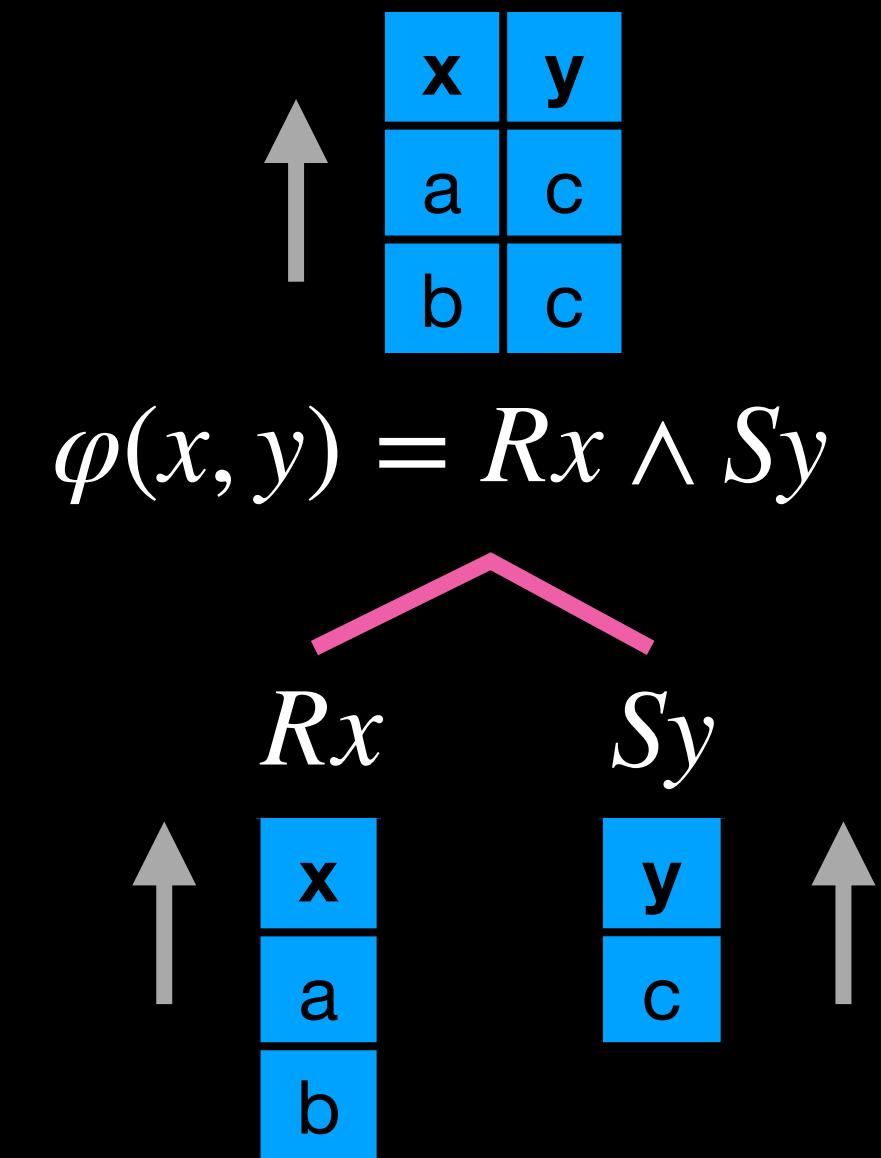
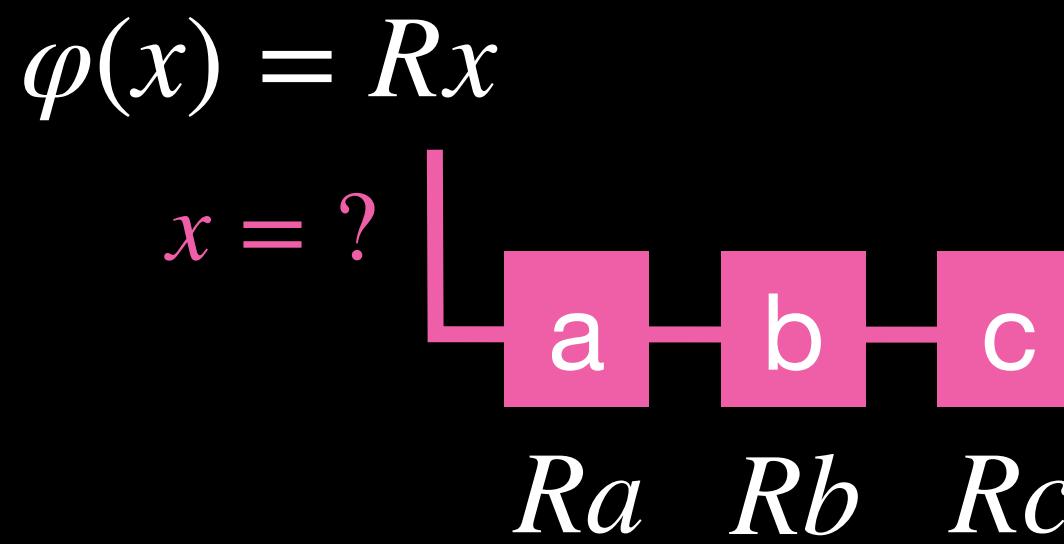
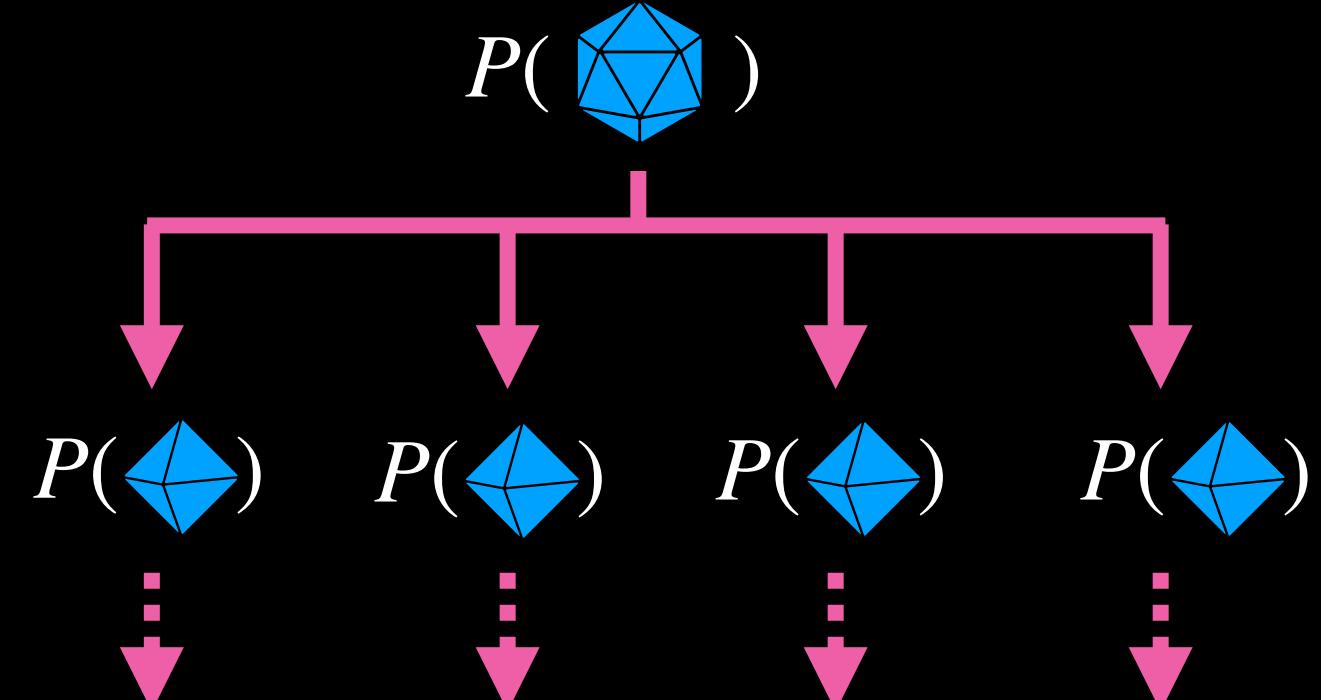
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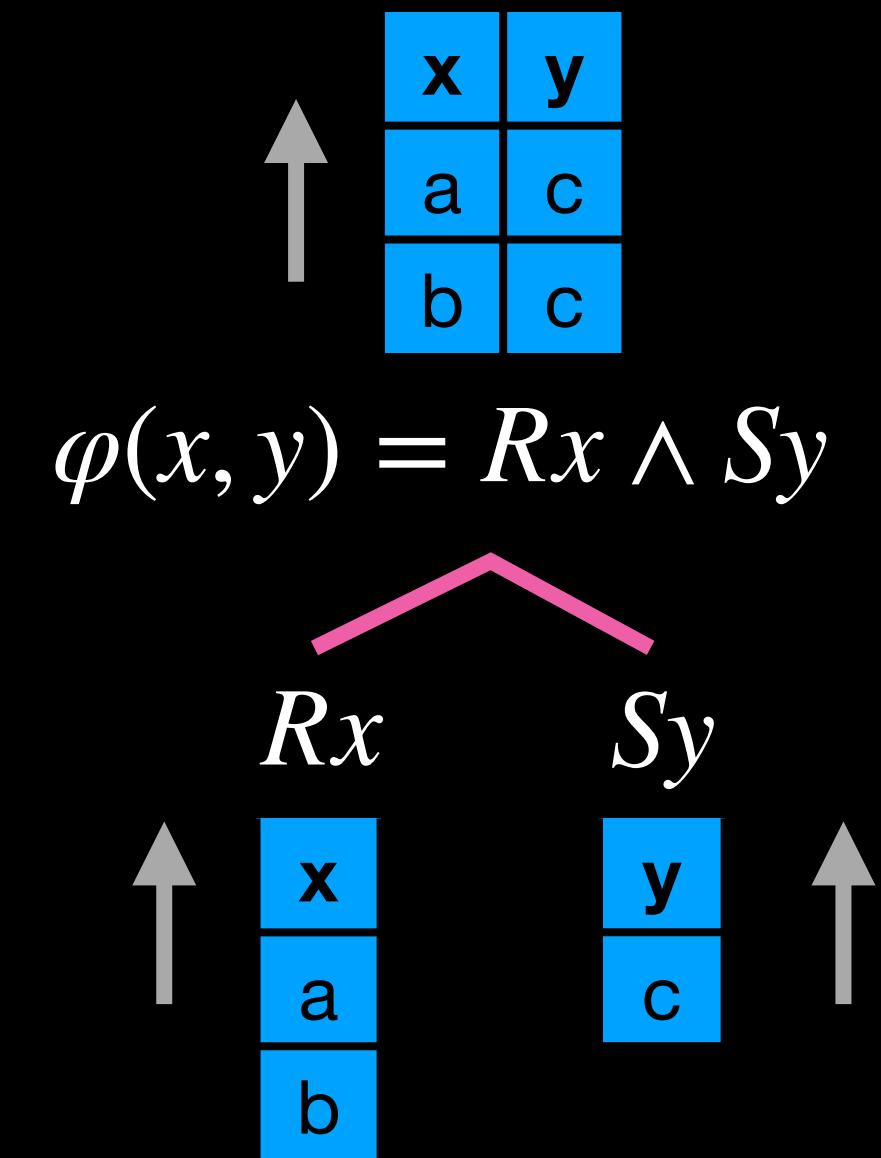
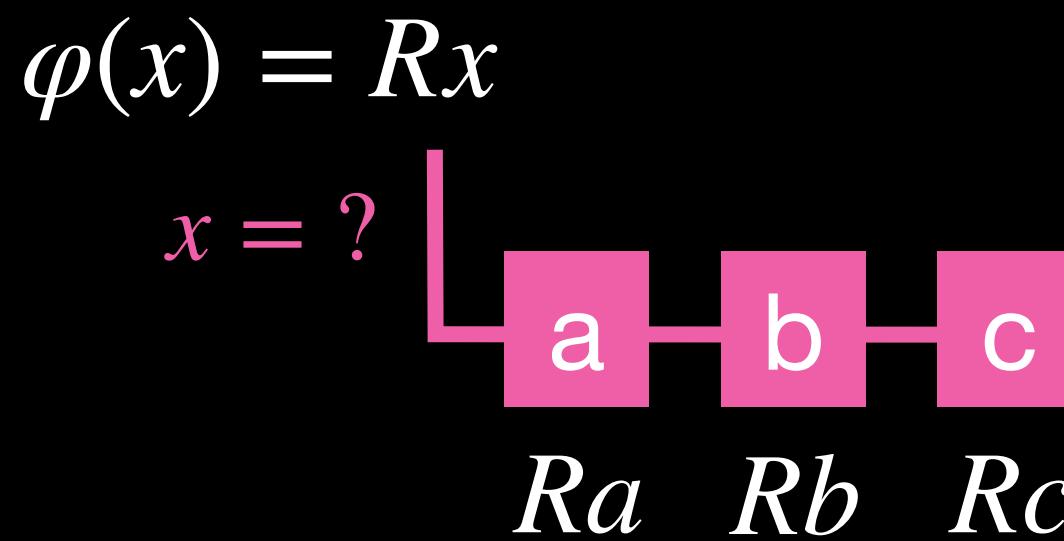
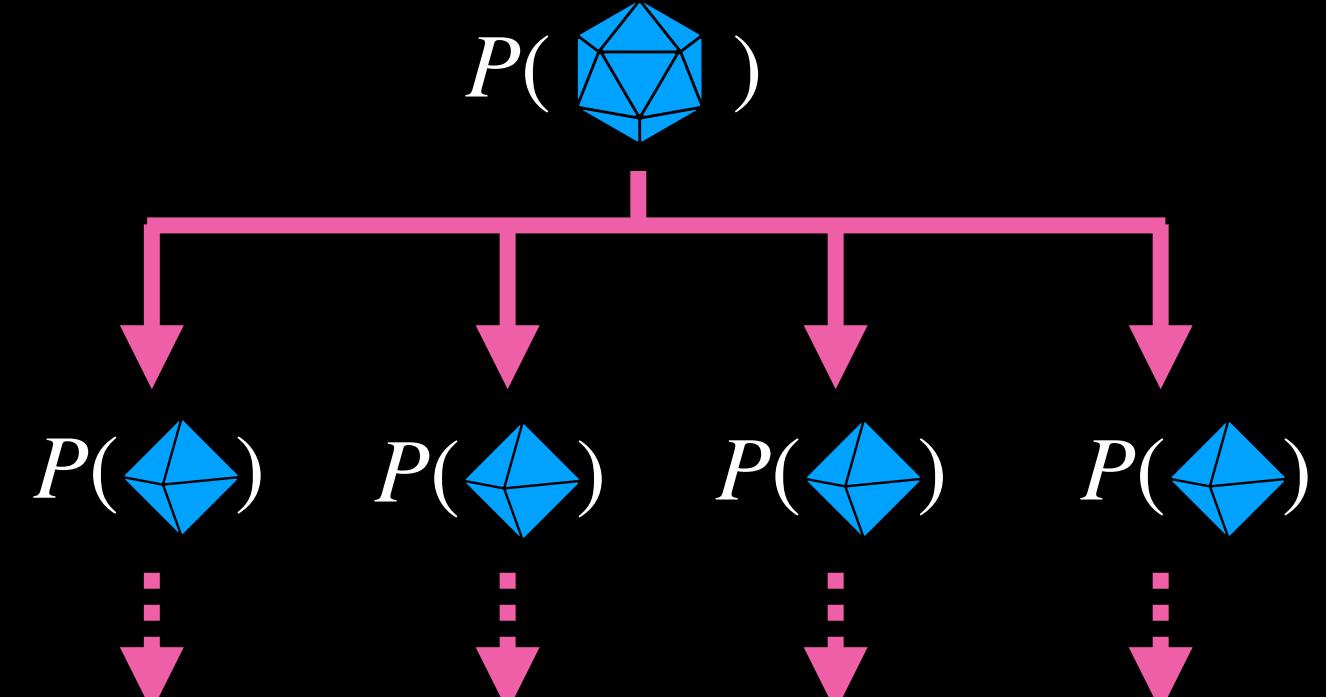


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...on tuple-independent  
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Query is q-hierarchical  $\Rightarrow$   
Dynamic Enumeration with  
 $O(1)$  delay and  $O(1)$  updates

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...on tuple-independent  
probabilistic DB's

Query is q-hierarchical  $\Rightarrow$

Dynamic Ranked Enumeration  
with  $O(\log n)$  delay and  $O(\log n)$  updates

# Ranked Enumeration for non-repeating CQs

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**Dichotomy:**

[Berkholz, Keppeler, Schweikardt 2017]

...on deterministic DB's

Non-repeating CQs

# Ranked Enumeration for non-repeating CQs

## Dichotomy:

[Berkholz, Keppeler, Schweikardt 2017]

...on **deterministic** DB's

If the OMv-conjecture holds:

Query is not q-hierarchical  $\Rightarrow$

Dynamic Enumeration with  
less than  $O(n^{1/2})$  delay and  $O(n^{1/2})$  updates  
is **impossible**

# Ranked Enumeration for non-repeating CQs

## Dichotomy:

[Berkholz, Keppeler, Schweikardt 2017]

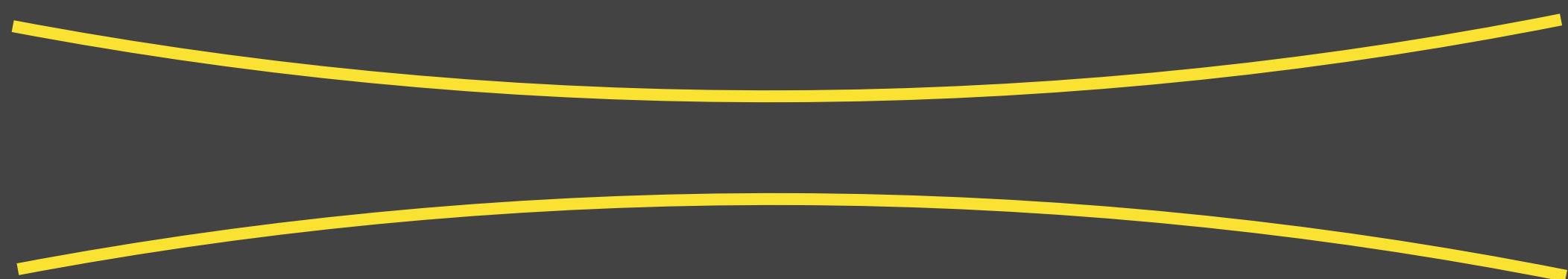
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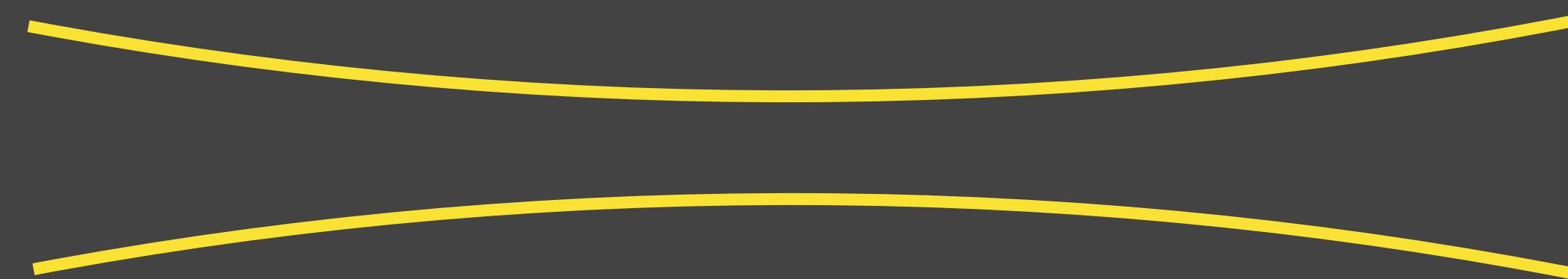
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Non-repeating CQs

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# Probabilistic Databases under Updates



Boolean Query Evaluation  
and Ranked Enumeration

Christoph Berkholz, Maximilian Merz

Talk at AIMoTh 2022, based on the PODS 2021 paper



# Probabilistic Databases under Updates



- ✓ Boolean Query Evaluation
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